American University of Beirut

STAT 210: Elementary Statistics for Sciences

2022–2023 Fall

Siamak Taati

Chapter 4
Probability
Part 3

Probability

Conditional probability

Once we learn some partial information about the outcome of a random experiment, we have to revise the probabilities assigned to events.

The revision of the probabilities must be done in a consistent and impartial fashion.

The revised probabilities in light of partial information are called conditional probabilities.

Conditional probability

Example (Sample of 50 birds)

	gray	pink	red	total
F	0	7	15	22
M	2	11	15	28
total	2	18	30	50

Contingency table for <u>sex</u> vs. <u>beak color</u>

Conditional probability

Example (Sample of 50 birds)

	gray	pink	red	total
F	0	7	15	22
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Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

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We pick one of the birds uniformly at random. [i.e., all equally likely]

Probability model of the experiment:

$$\boxed{\Omega := \text{collection of all the 50 birds}}$$

$$\boxed{\mathbb{P}(a) = 1/50 \qquad \text{for each bird } a}$$

Conditional probability

Example (Sample of 50 birds)

	gray	pink	red	total
F	0	7	15	22
M	2	11	15	28
total	2	18	30	50

Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q What is the chance that the picked bird is female?

Conditional probability

Example (Sample of 50 birds)

	gray	pink	red	total
F	0	7	15	22
M	2	11	15	28
total	2	18	30	50

Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q What is the chance that the picked bird is female?

A Since all the birds are equally likely to be picked,

$$\mathbb{P}(\text{female}) = \frac{\#\text{ of female birds}}{\text{total }\#\text{ of birds}} = \frac{22}{50} = \boxed{0.44}$$

Conditional probability

Example (Sample of 50 birds)

	gray	pink	red	total
F	0	7	15	22
M	2	11	15	28
total	2	18	30	50

Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q What is the chance that the picked bird has a pink beak?

Conditional probability

Example (Sample of 50 birds)

	gray	pink	red	total
F	0	7	15	22
M	2	11	15	28
total	2	18	30	50

Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q What is the chance that the picked bird has a pink beak?

A Since all the birds are equally likely to be picked,

$$\mathbb{P}(\text{pink beak}) = \frac{\#\text{ of pink-beaked birds}}{\text{total }\#\text{ of birds}} = \frac{18}{50} = \boxed{0.36}$$

Conditional probability

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	gray	pink	red	total
F	0	7	15	22
M	2	11	15	28
total	2	18	30	50

Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q What is the chance that the picked bird is female and has a pink beak?

Conditional probability

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	gray	pink	red	total
F	0	7	15	22
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total	2	18	30	50

Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q What is the chance that the picked bird is female and has a pink beak?

A Since all the birds are equally likely to be picked,

$$\mathbb{P}(\text{female and pink beak}) = \frac{\#\text{ of pink-beaked female birds}}{\text{total }\#\text{ of birds}} = \frac{7}{50} = \boxed{0.14}$$

Conditional probability

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Contingency table for sex vs. beak color

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Q If the picked bird has a pink beak, what is the chance that it is female?

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	gray	pink	red	total
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Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q If the picked bird has a pink beak, what is the chance that it is female?

A1 Since all the pink-beaked birds are equally likely to be picked,

$$\mathbb{P}(\text{female} \mid \text{pink beak}) = \frac{\#\text{ of female pink-beaked birds}}{\text{total }\#\text{ of pink-beaked birds}} = \frac{7}{18} \approx \boxed{0.389}$$

(Read: conditional probability that the bird is female given that it has a pink beak.)

Conditional probability

Example (Sample of 50 birds)

	gray	pink	red	total
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Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q If the picked bird has a pink beak, what is the chance that it is female?

A2 We already know $\mathbb{P}(\text{female and pink beak})$ and $\mathbb{P}(\text{pink beak})$:

$$\mathbb{P}(\text{female} \mid \text{pink beak}) = \frac{\mathbb{P}(\text{female and pink beak})}{\mathbb{P}(\text{pink beak})} = \frac{0.14}{0.36} \approx \boxed{0.389}$$

(Read: conditional probability that the bird is female given that it has a pink beak.)

Conditional probability

The conditional probability of an event A given another event C is the (consistently and impartially) revised probability that we should assign to A once we learn that C has happened.

It is denoted by $\mathbb{P}(A \mid C)$ and can be calculated as

$$\mathbb{P}(A \mid C) := \frac{\mathbb{P}(A \text{ and } C)}{\mathbb{P}(C)}$$

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If all the outcomes are equally likely, then

$$\mathbb{P}(A \mid C) = \frac{\# \text{ of outcomes that realize both } A \text{ and } C}{\# \text{ of outcomes that realize } C}$$

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Remark

The conditional probability $\mathbb{P}(A \mid C)$ is meaningful only if $\mathbb{P}(C) \neq 0$.

Conditional probability and independence

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Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q Are the two events $\langle$female$\rangle$ and $\langle$pink beak$\rangle$ independent?

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Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q Are the two events ⟨female⟩ and ⟨pink beak⟩ independent?

A1 We know

$$\mathbb{P}(\text{female}) = \frac{22}{50}\,,\ \mathbb{P}(\text{pink beak}) = \frac{18}{50}\,,\ \mathbb{P}(\text{female and pink beak}) = \frac{7}{50}$$

Since

$$\mathbb{P}(\text{female})\,\mathbb{P}(\text{pink beak}) = \frac{22}{50} \times \frac{18}{50} \neq \frac{7}{50} = \mathbb{P}(\text{female and pink beak})\,,$$

the two events are not independent.

Conditional probability and independence

Example (Sample of 50 birds)

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Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q Are the two events ⟨female⟩ and ⟨pink beak⟩ independent?

A2 We know

$$\mathbb{P}(\text{female}) = {}^{22}/_{50}\ , \qquad \mathbb{P}(\text{female} \mid \text{pink beak}) = {}^{7}/_{18}$$

Since ${}^{7}/_{18} < {}^{22}/_{50}$, the two events are not independent.

The pinkness of the beak carries some information about whether the bird is female or not: it makes it less likely.

Conditional probability and independence

Example (Sample of 50 birds)

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Contingency table for sex vs. beak color

We pick one of the birds uniformly at random. [i.e., all equally likely]

Q Are the two events ⟨female⟩ and ⟨pink beak⟩ independent?

A3 We know

$$\mathbb{P}(\text{pink beak}) = {}^{18}/_{50}\,, \qquad \mathbb{P}(\text{pink beak} \mid \text{female}) = {}^{7}/_{22}$$

Since ${}^{7}/_{22} < {}^{18}/_{50}$, the two events are not independent.

Knowing that the bird is female carries information about whether its beak is pink or not: it makes it less likely.

Conditional probability and independence

If two events A and B have non-zero probabilities, then any of the following equivalent conditions means that A and B are independent:

(1) $\mathbb{P}(A \text{ and } B) = \mathbb{P}(A)\,\mathbb{P}(B)$.

(2) $\mathbb{P}(A \mid B) = \mathbb{P}(A)$.

(3) $\mathbb{P}(B \mid A) = \mathbb{P}(B)$.

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- If $\mathbb{P}(A \text{ and } B) < \mathbb{P}(A)\,\mathbb{P}(B)$, we say that A and B are negatively associated.

  [The occurrence of one makes the other less likely.]

- If $\mathbb{P}(A \text{ and } B) > \mathbb{P}(A)\,\mathbb{P}(B)$, we say that A and B are positively associated.

  [The occurrence of one makes the other more likely.]

Conditional probability

For two independent events A and B, we have

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In general, we have the following:

Chain rule of conditional probabilities

- For every two events A and B, we have

$$\boxed{\mathbb{P}(A \text{ and } B) = \mathbb{P}(A)\,\mathbb{P}(B \mid A)}$$

(provided $\mathbb{P}(A) \neq 0$.)

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  (provided $\mathbb{P}(A) \neq 0$.)

- For every three events A, B and C, we have

$$\boxed{\mathbb{P}(A \text{ and } B \text{ and } C) = \mathbb{P}(A)\,\mathbb{P}(B \mid A)\,\mathbb{P}(C \mid A \text{ and } B)}$$

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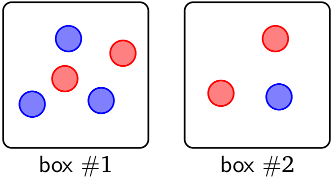
- ... (similarly for any number of events)

Multi-stage experiments

Example (Boxes and balls)

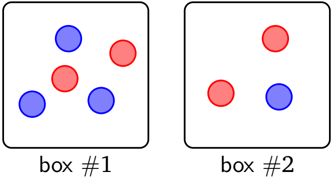


We have two boxes with red and blue balls in them.

- In box #1, there are 2 red balls and 3 blue balls.
- In box #2, there are 2 red balls and 1 blue balls.

Multi-stage experiments

Example (Boxes and balls)



We perform the following experiment:

(1) We select a box uniformly at random.

(2) We pick a ball from the selected box uniformly at random.

Multi-stage experiments

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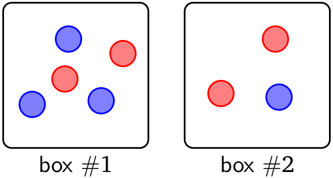

We perform the following experiment:

(1) We select a box uniformly at random.

(2) We pick a ball from the selected box uniformly at random.

(Q) What is the chance that the picked ball is blue?

Multi-stage experiments

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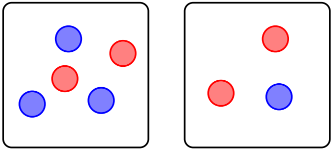


We perform the following experiment:

(1) We select a box uniformly at random.

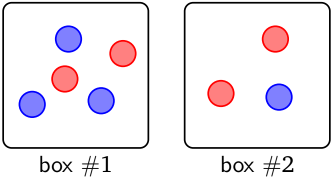
(2) We pick a ball from the selected box uniformly at random.

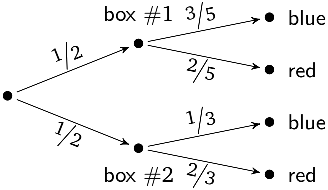
Q What is the chance that the picked ball is blue?

Warning: The 8 balls are not all equally likely to be picked.

Multi-stage experiments

Example (Boxes and balls)





We perform the following experiment:

(1) We select a box uniformly at random.

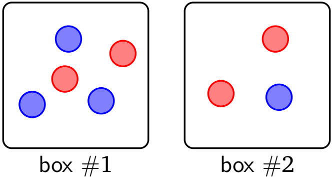
(2) We pick a ball from the selected box uniformly at random.

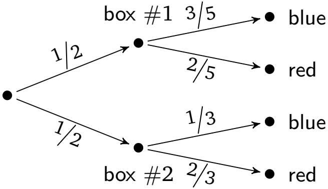
Q What is the chance that the picked ball is blue?

A Consider the tree of possibilities.

Multi-stage experiments

Example (Boxes and balls)





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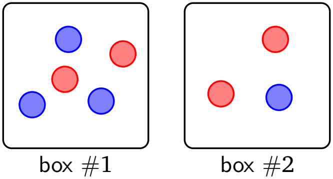
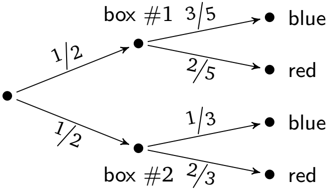




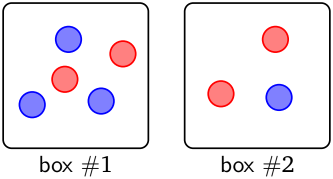
Q What is the chance that the picked ball is <u>blue</u>?

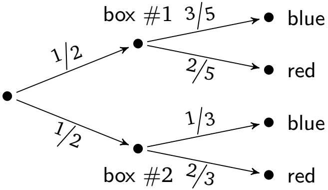
A Consider the tree of possibilities.

$$\begin{aligned}\mathbb{P}(\text{blue}) &= \mathbb{P}(\text{box \#1 and blue}) + \mathbb{P}(\text{box \#2 and blue}) \\ &= (1/2) \times (3/5) + (1/2) \times (1/3) = \boxed{14/30}\,.\end{aligned}$$

Multi-stage experiments

Example (Boxes and balls)





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$$\begin{aligned}\mathbb{P}(\text{blue}) &= \mathbb{P}(\text{box \#1 and blue}) + \mathbb{P}(\text{box \#2 and blue}) \\ &= \mathbb{P}(\text{box \#1})\,\mathbb{P}(\text{blue} \mid \text{box \#1}) + \mathbb{P}(\text{box \#2})\,\mathbb{P}(\text{blue} \mid \text{box \#2}) \\ &= (1/2) \times (3/5) + (1/2) \times (1/3) = \boxed{14/30}\,.\end{aligned}$$

Conditional probability

In order to calculate the probability of an event A, it is sometimes helpful to break the possibilities based on whether another event C has happened or not.

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Principle of total probability

Consider an event A.

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Principle of total probability

Consider an event A.

- If C is any other event, then

$$\boxed{\mathbb{P}(A) = \mathbb{P}(A \text{ and } C) + \mathbb{P}(A \text{ and not } C)}$$

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$$= \mathbb{P}(C)\,\mathbb{P}(A \mid C) + \mathbb{P}(\text{not } C)\,\mathbb{P}(A \mid \text{not } C)$$

- If C_1, C_2 and C_3 are <u>mutually exclusive</u> and together <u>exhaust all the possibilities</u>, then

$$\boxed{\mathbb{P}(A) = \mathbb{P}(A \text{ and } C_1) + \mathbb{P}(A \text{ and } C_2) + \mathbb{P}(A \text{ and } C_3)}$$
$$= \mathbb{P}(C_1)\,\mathbb{P}(A \mid C_1) + \mathbb{P}(C_2)\,\mathbb{P}(A \mid C_2) + \mathbb{P}(C_3)\,\mathbb{P}(A \mid C_3)$$

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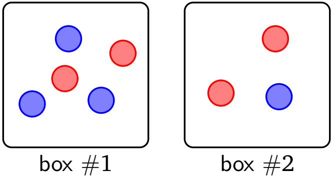
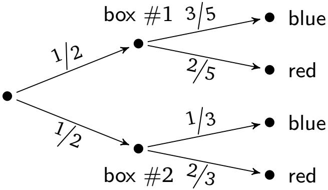
- ...

Reverse conditional probabilities

Example (Boxes and balls)





A friend performs the experiment while we are not looking. She then shows us the ball.

Reverse conditional probabilities

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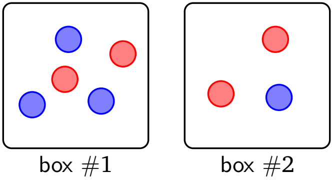

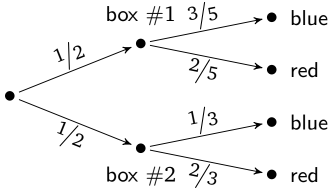

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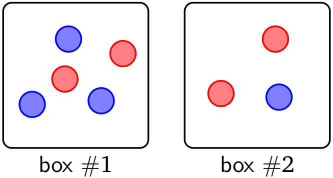
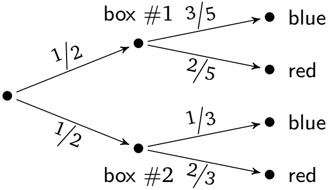




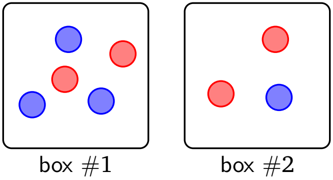
A friend performs the experiment while we are not looking. She then shows us the ball.

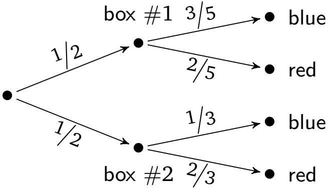
Q If the picked ball is blue, what is the chance that box $\#1$ was chosen?

Note: We know $\mathbb{P}(\text{blue} \mid \text{box } \#1) = {}^3\!/_5$.
We want $\mathbb{P}(\text{box } \#1 \mid \text{blue})$.

Reverse conditional probabilities

Example (Boxes and balls)





A friend performs the experiment while we are not looking. She then shows us the ball.

Q If the picked ball is blue, what is the chance that box #1 was chosen?

A We already know $\mathbb{P}(\text{blue}) = {}^{14}/_{30}$. Therefore,

$$\mathbb{P}(\text{box } \#1 \mid \text{blue}) = \frac{\mathbb{P}(\text{box } \#1 \text{ and blue})}{\mathbb{P}(\text{blue})} = \frac{({}^{1}/_{2}) \times ({}^{3}/_{5})}{{}^{14}/_{30}} = \boxed{{}^{9}/_{14}}\,.$$

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In order to calculate $\mathbb{P}(E)$, it is often helpful to use the principle of total probability.

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Guess before calculating!

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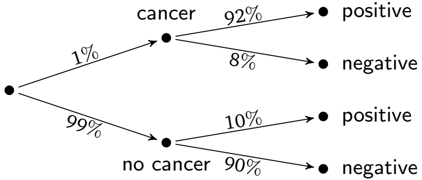

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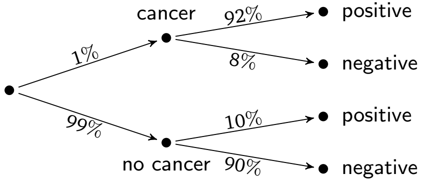

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$$\mathbb{P}(\text{cancer} \mid \text{positive}) = \frac{\mathbb{P}(\text{cancer and positive})}{\mathbb{P}(\text{positive})}$$

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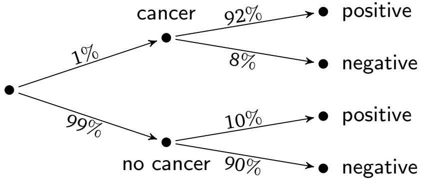

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$$\mathbb{P}(\text{cancer} \mid \text{positive}) = \frac{\mathbb{P}(\text{cancer and positive})}{\mathbb{P}(\text{positive})}$$
$$= \frac{0.01 \times 0.92}{0.01 \times 0.92 + 0.99 \times 0.1} = \frac{92}{1082} \approx \boxed{8.50\%}$$