

American University of Beirut
STAT 210: Elementary Statistics for Sciences
2022–2023 Fall

Siamak Taati

Chapter 6
Continuous random variables
Part 3

Continuous random variables

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

Normal distribution: some exercises

Exercise 1

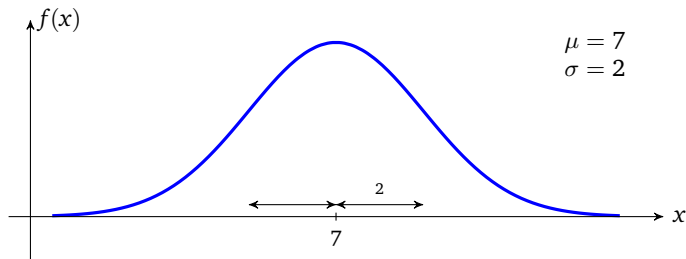
Suppose X is a normal RV with mean 7 and variance 4.

Q1 What is the probability that $X \geq 8$?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

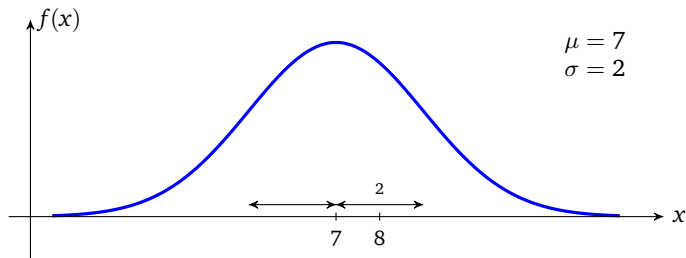


Q1 What is the probability that $X \geq 8$?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

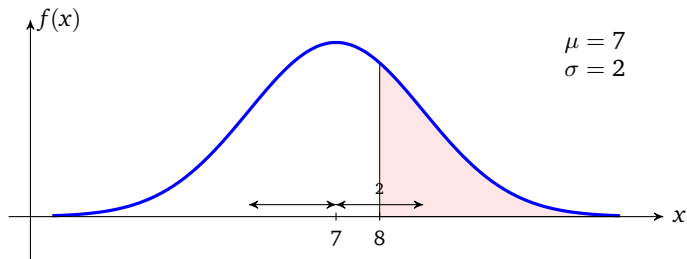


Q1 What is the probability that $X \geq 8$?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

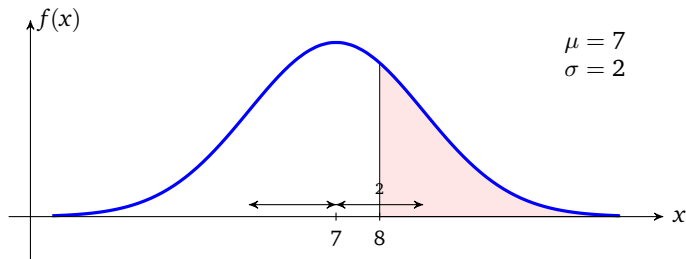


Q1 What is the probability that $X \geq 8$?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.



Q1 What is the probability that $X \geq 8$?

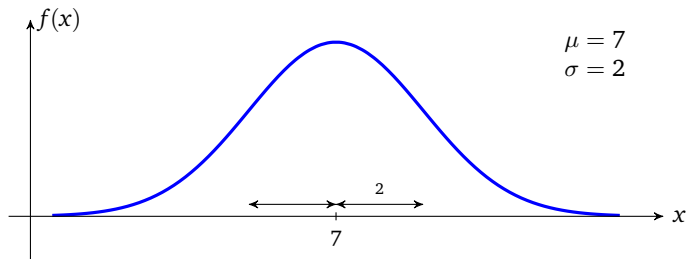
A Using the applet for normal distribution,

$$\mathbb{P}(X \geq 8) \approx \boxed{0.30854}.$$

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

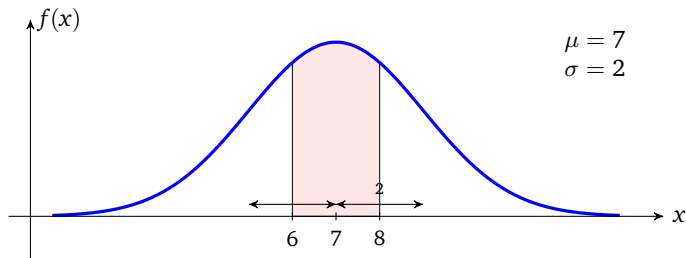


(Q2) What is the probability that $6 \leq X \leq 8$?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

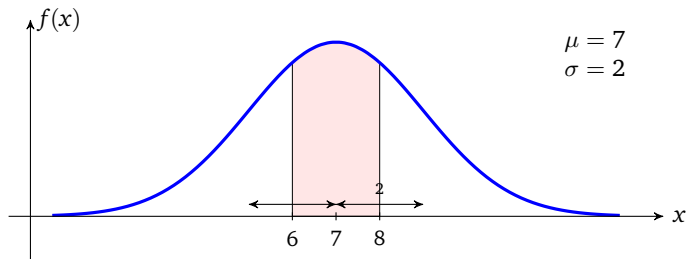


(Q2) What is the probability that $6 \leq X \leq 8$?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.



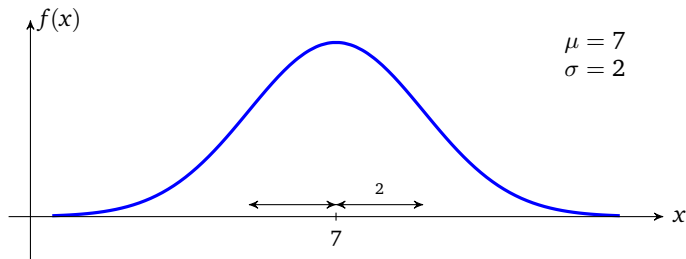
Q2 What is the probability that $6 \leq X \leq 8$?

A
$$\begin{aligned}\mathbb{P}(6 \leq X \leq 8) &= 1 - \mathbb{P}(X < 6) - \mathbb{P}(X > 8) \\ &= 1 - 2\mathbb{P}(X > 8) \quad \text{(by symmetry)} \\ &\approx 1 - 2 \times 0.30854 = \boxed{0.38292}.\end{aligned}$$

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

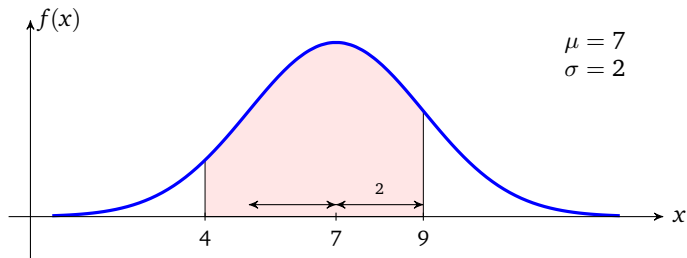


Q3 What is the probability that $4 \leq X \leq 9$?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

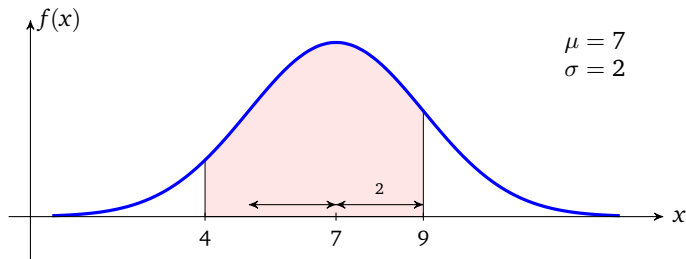


Q3 What is the probability that $4 \leq X \leq 9$?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.



Q3 What is the probability that $4 \leq X \leq 9$?

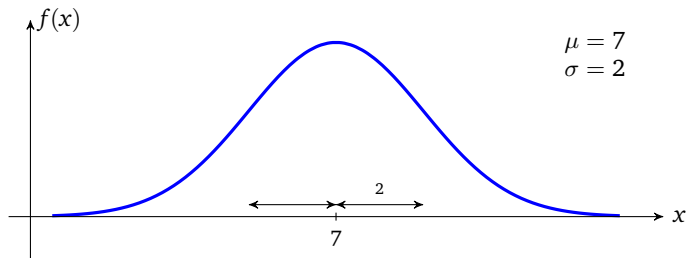
A

$$\begin{aligned}\mathbb{P}(4 \leq X \leq 9) &= \mathbb{P}(X \leq 9) - \mathbb{P}(X < 4) \\ &\approx 0.84134 - 0.06681 \quad (\text{using the applet}) \\ &= \boxed{0.77453}.\end{aligned}$$

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

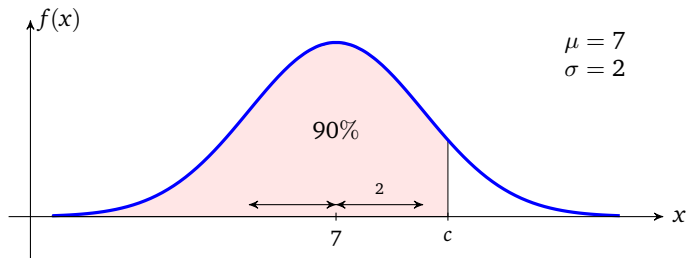


(Q4) For which value of c , we have $X \leq c$ with probability 90%?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

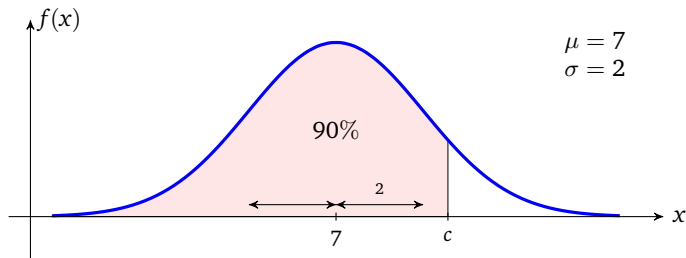


Q4 For which value of c , we have $X \leq c$ with probability 90%?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.



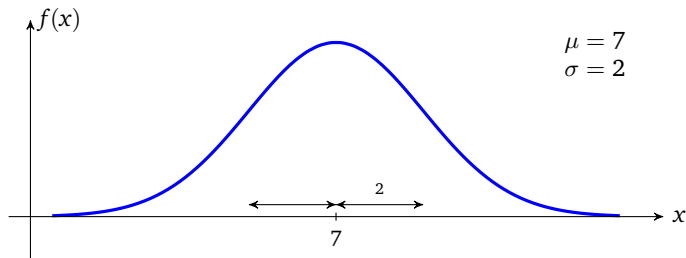
Q4 For which value of c , we have $X \leq c$ with probability 90%?

A We want c such that $\mathbb{P}(X \leq c) = 0.9$. Using the applet for normal distribution, we find $c \approx \boxed{9.5631}$.

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

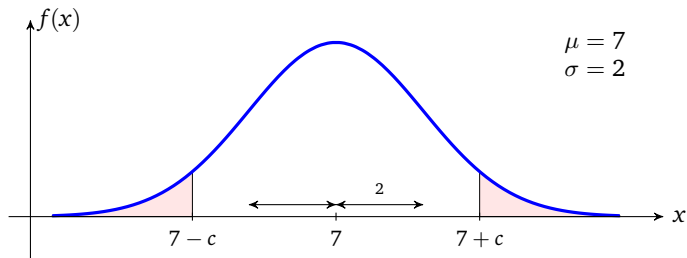


(Q5) For which value of c , we have $|X - 7| > c$ with probability 10%?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

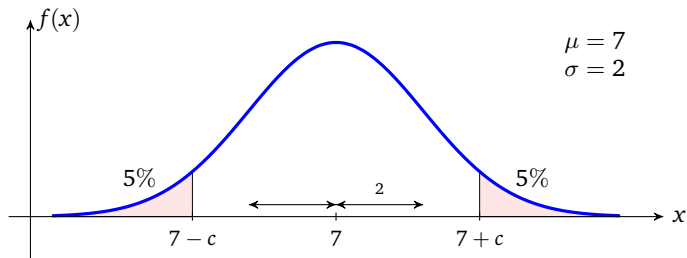


(Q5) For which value of c , we have $|X - 7| > c$ with probability 10%?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.

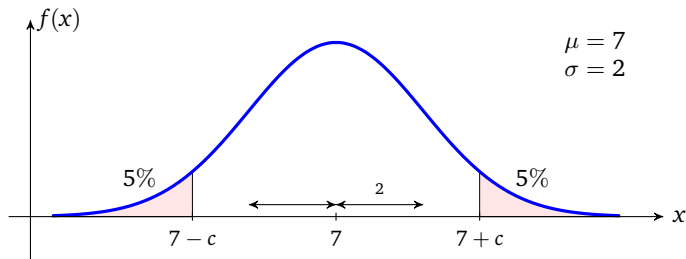


(Q5) For which value of c , we have $|X - 7| > c$ with probability 10%?

Normal distribution: some exercises

Exercise 1

Suppose X is a normal RV with mean 7 and variance 4.



Q5 For which value of c , we have $|X - 7| > c$ with probability 10%?

A We want c such that $\mathbb{P}(|X - 7| > c) = 0.1$. Note that

$$\begin{aligned} 0.1 &= \mathbb{P}(|X - 7| > c) = \mathbb{P}(X > 7 + c) + \mathbb{P}(X < 7 - c) \\ &= 2\mathbb{P}(X > 7 + c) \quad (\text{by symmetry}), \end{aligned}$$

therefore $\mathbb{P}(X > 7 + c) = 0.05$. Using the applet for normal distribution, we have $7 + c \approx 10.28971$, hence $c \approx \boxed{3.28971}$.

Normal distribution: some exercises

Exercise 2

Suppose X is a normal RV with mean μ and standard deviation σ .

Normal distribution: some exercises

Exercise 2

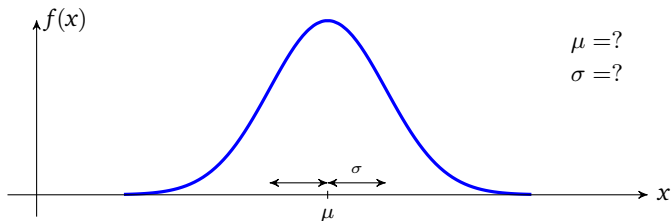
Suppose X is a normal RV with mean μ and standard deviation σ .

- Q If $|X - \mu| \leq 2$ with probability 95%, what is the standard deviation of X ?

Normal distribution: some exercises

Exercise 2

Suppose X is a normal RV with mean μ and standard deviation σ .

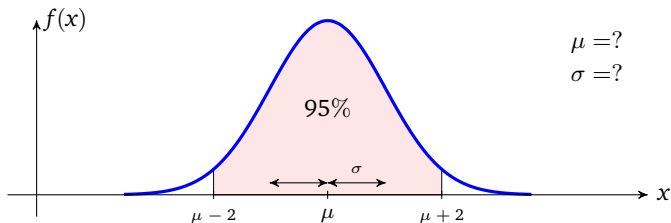


- Q If $|X - \mu| \leq 2$ with probability 95%, what is the standard deviation of X ?

Normal distribution: some exercises

Exercise 2

Suppose X is a normal RV with mean μ and standard deviation σ .

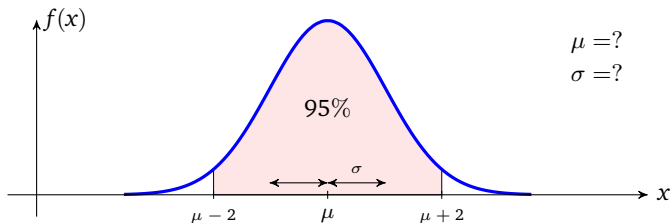


- Q If $|X - \mu| \leq 2$ with probability 95%, what is the standard deviation of X ?

Normal distribution: some exercises

Exercise 2

Suppose X is a normal RV with mean μ and standard deviation σ .



Q If $|X - \mu| \leq 2$ with probability 95%, what is the standard deviation of X ?

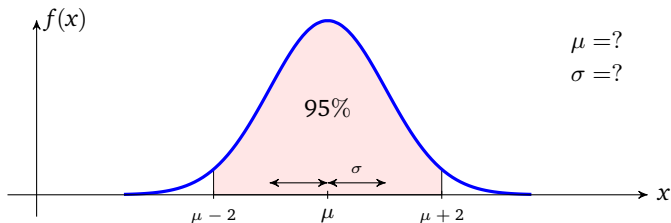
A Let $Z := (X - \mu)/\sigma$ be the standardized version of X . We know that

$$0.95 = \mathbb{P}(\mu - 2 \leq X \leq \mu + 2)$$

Normal distribution: some exercises

Exercise 2

Suppose X is a normal RV with mean μ and standard deviation σ .



Q If $|X - \mu| \leq 2$ with probability 95%, what is the standard deviation of X ?

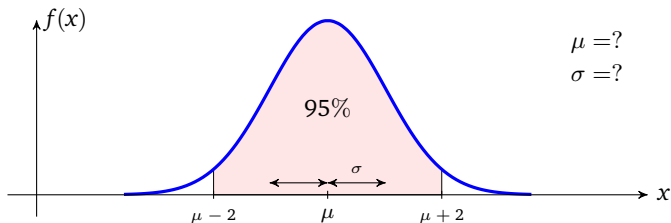
A Let $Z := (X - \mu)/\sigma$ be the standardized version of X . We know that

$$0.95 = \mathbb{P}(\mu - 2 \leq X \leq \mu + 2) = \mathbb{P}(-2/\sigma \leq (X - \mu)/\sigma \leq 2/\sigma)$$

Normal distribution: some exercises

Exercise 2

Suppose X is a normal RV with mean μ and standard deviation σ .



Q If $|X - \mu| \leq 2$ with probability 95%, what is the standard deviation of X ?

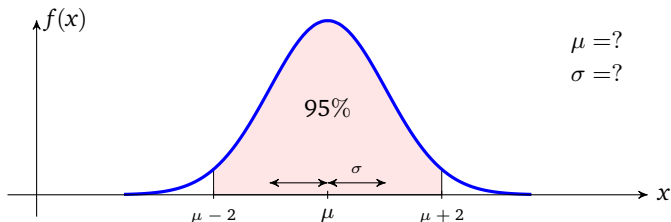
A Let $Z := (X - \mu)/\sigma$ be the standardized version of X . We know that

$$\begin{aligned} 0.95 &= \mathbb{P}(\mu - 2 \leq X \leq \mu + 2) = \mathbb{P}(-2/\sigma \leq (X - \mu)/\sigma \leq 2/\sigma) \\ &= \mathbb{P}(-2/\sigma \leq Z \leq 2/\sigma). \end{aligned}$$

Normal distribution: some exercises

Exercise 2

Suppose X is a normal RV with mean μ and standard deviation σ .



Q If $|X - \mu| \leq 2$ with probability 95%, what is the standard deviation of X ?

A Let $Z := (X - \mu)/\sigma$ be the standardized version of X . We know that

$$0.95 = \mathbb{P}(\mu - 2 \leq X \leq \mu + 2) = \mathbb{P}(-2/\sigma \leq (X - \mu)/\sigma \leq 2/\sigma)$$
$$= \mathbb{P}(-2/\sigma \leq Z \leq 2/\sigma).$$

Using the applet for normal distribution, we find $2/\sigma \approx 1.95996$,
hence $\sigma \approx 2/1.95996 \approx \boxed{1.02043}$.

Normal distribution: some exercises

Exercise 3

Suppose X is a normal RV with mean μ and standard deviation 3.5.

Normal distribution: some exercises

Exercise 3

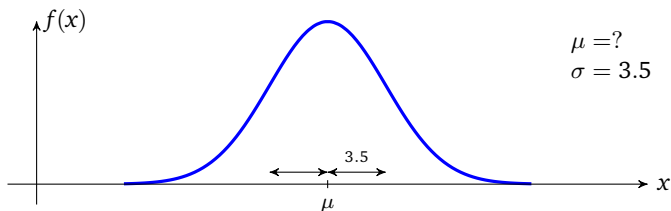
Suppose X is a normal RV with mean μ and standard deviation 3.5.

- ① If $X > 21$ with probability at most 4%, what can be said about the mean of X ?

Normal distribution: some exercises

Exercise 3

Suppose X is a normal RV with mean μ and standard deviation 3.5.

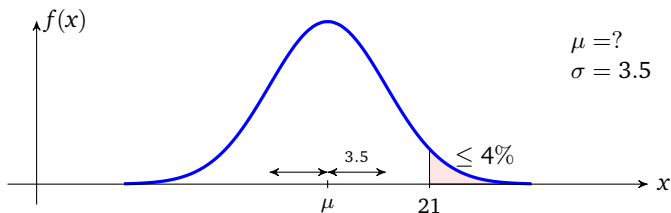


- ① Q If $X > 21$ with probability at most 4%, what can be said about the mean of X ?

Normal distribution: some exercises

Exercise 3

Suppose X is a normal RV with mean μ and standard deviation 3.5.

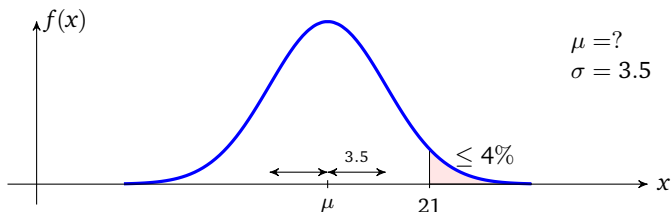


- Q If $X > 21$ with probability at most 4%, what can be said about the mean of X ?

Normal distribution: some exercises

Exercise 3

Suppose X is a normal RV with mean μ and standard deviation 3.5.



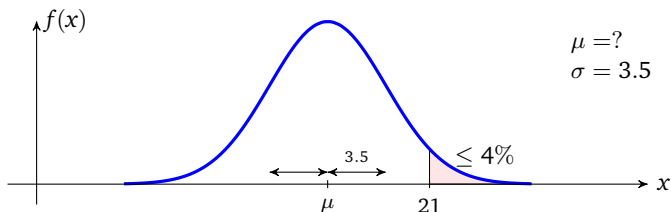
Q If $X > 21$ with probability at most 4%, what can be said about the mean of X ?

A Let $Z := (X - \mu)/3.5$ be the standardized version of X . We know that $0.04 \geq \mathbb{P}(X > 21)$

Normal distribution: some exercises

Exercise 3

Suppose X is a normal RV with mean μ and standard deviation 3.5.



$$\begin{aligned}\mu &=? \\ \sigma &= 3.5\end{aligned}$$

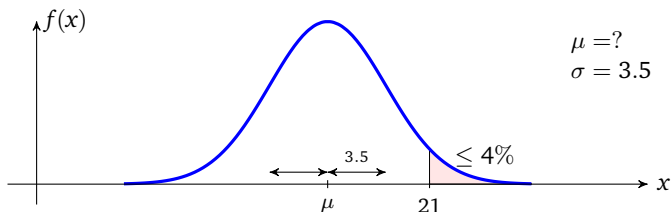
Q If $X > 21$ with probability at most 4%, what can be said about the mean of X ?

A Let $Z := (X - \mu)/3.5$ be the standardized version of X . We know that $0.04 \geq \mathbb{P}(X > 21) = \mathbb{P}((X - \mu)/3.5 > (21 - \mu)/3.5)$

Normal distribution: some exercises

Exercise 3

Suppose X is a normal RV with mean μ and standard deviation 3.5.



Q If $X > 21$ with probability at most 4%, what can be said about the mean of X ?

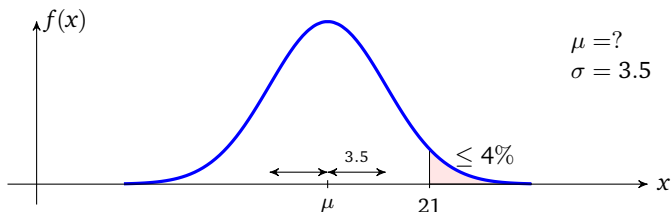
A Let $Z := (X - \mu)/3.5$ be the standardized version of X . We know that

$$0.04 \geq \mathbb{P}(X > 21) = \mathbb{P}((X - \mu)/3.5 > (21 - \mu)/3.5)$$
$$= \mathbb{P}(Z > (21 - \mu)/3.5) .$$

Normal distribution: some exercises

Exercise 3

Suppose X is a normal RV with mean μ and standard deviation 3.5.



Q If $X > 21$ with probability at most 4%, what can be said about the mean of X ?

A Let $Z := (X - \mu)/3.5$ be the standardized version of X . We know that

$$0.04 \geq \mathbb{P}(X > 21) = \mathbb{P}((X - \mu)/3.5 > (21 - \mu)/3.5)$$
$$= \mathbb{P}(Z > (21 - \mu)/3.5) .$$

Using the applet for normal distribution, we find

$(21 - \mu)/3.5 \gtrapprox 1.75069$, hence $\mu \lesssim \boxed{14.872585}$.