American University of Beirut

STAT 210: Elementary Statistics for Sciences

2022–2023 Fall

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Chapter 8

Estimation of mean and proportion

Part 1

Estimation of population parameters

Estimation of population parameters

Last week we saw how we can use information about the population (population distribution and/or population parameters) to describe the distribution of a sample mean or a sample proportion.

In practice, we rarely have access to the population parameters. Remember: the main objective of Statistics is to learn about populations using statistical evidence.

Estimation of population parameters

We now discuss the problem of estimating population parameters on the basis of statistical data.

Namely, we would like to use sample data to estimate:

- population mean μ
- population standard deviation σ (discussed later in the course)
- population proportion p
- ...

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We focus on the case where the data comes from a random sample. If the sample is not (entirely) random, then the problem becomes more complicated.

Estimation of population parameters

Example (Weight of gorillas)

Researchers studying gorillas would like to know average weight of an adult gorilla (i.e., the population mean).

Since they do not have access to the entire population, they decide to <u>take a sample</u> of gorillas, <u>measure</u> their weights, and use this data as evidence to <u>estimate</u> the population mean.

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A Comparing gorillas with orangutans, we may guess that the population distribution is bimodal. Other than that, we know nothing.

Estimation of population parameters

Example (Weight of gorillas)

The researchers take a sample of 70 adult gorillas and measure their weights (in Kg):

w_1	w_2	w_3	w_4	$\cdots$	w_{70}	$\overline{w}$	s
150.81	92.25	92.00	97.98	$\cdots$	153.89	128.07	34.286

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Disadvantage

The single value 128.07 Kg does not give any indication of the accuracy and reliability of the estimate.

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A2 A better type of estimate:

$$\boxed{128.07 \pm 8.18\,\text{Kg} \quad \text{(with 95\% confidence)}}$$

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It includes an indication of the accuracy and reliability of the estimate.

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Q What is a good estimate for the <u>population mean</u>?

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This is called an interval estimate (or a confidence interval).

Q1 What exactly does such an estimate mean?

Q2 How can we calculate such an estimate?

Estimation of population parameters

Example (Weight of gorillas)

Consider the interval estimate

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for the population mean weight of gorillas.

Estimation of population parameters

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- $E := 8.18$ is called the margin of error.

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- 95% is called the confidence level.

The same sample can be used to find other confidence intervals:

$$128.07 \pm 6.83\,\text{Kg} \quad \text{(with 90\% confidence)} \tag{1}$$

$$128.07 \pm 8.18\,\text{Kg} \quad \text{(with 95\% confidence)} \tag{2}$$

$$128.07 \pm 10.86\,\text{Kg} \quad \text{(with 99\% confidence)} \tag{3}$$

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But there is a trade-off between the margin of error and the confidence level. We cannot improve both unless we have access to more statistical evidence (i.e., a larger sample).

Estimation of population parameters

Point estimate/estimator

A point estimate for a population parameter θ is a single value (calculated on the basis of a random sample) that is meant to approximate the true value of θ.

The procedure used to obtain a point estimate is called a point estimator.

The point estimator for a parameter θ is often denoted by $\widehat{\theta}$.

We often use the corresponding sample statistic as point estimator:

population parameter		point estimator	
μ	(population mean)	$\widehat{\mu} = \overline{X}$	(sample mean)
σ	(population stand. dev.)	$\widehat{\sigma} = S$	(sample stand. dev.)
p	(population proportion)	$\widehat{p}$	(sample proportion)
$\vdots$		$\vdots$	

Estimation of population parameters

Interval estimate/estimator

A 95% confidence interval for a population parameter θ is an interval of the form

$$\text{point estimate} \pm \text{margin of error}$$

(calculated on the basis of a random sample) that is meant to estimate the true value of θ.

The procedure used to obtain a confidence interval is called an interval estimator.

Estimation of population parameter

True value of a parameter vs. its estimate

- The true value of a population parameter θ is a fixed number, which is often unknown to us.
- The estimate for θ provided by a point/interval estimator varies from sample to sample. More precisely, since the sample values are random, so is the estimate provided by the estimator.

Estimation of population mean

Finding confidence intervals for the population mean

Suppose we want to estimate the population mean μ of a variable X based on a random sample of size n.

The procedure depends on the answers to the following questions:

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Suppose we want to estimate the population mean μ of a variable X based on a random sample of size n.

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Q1 Is the sample size large or small?

Q2 Do we know the population standard deviation σ?

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Suppose we want to estimate the population mean μ of a variable X based on a random sample of size n.

The procedure depends on the answers to the following questions:

Q1 Is the sample size large or small?

Q2 Do we know the population standard deviation σ?

Q3 Is the population (approximately) normally distributed?

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Finding confidence intervals for the population mean

Suppose we want to estimate the population mean μ of a variable X based on a random sample of size n.

We will consider the following scenarios:

1. σ known, normal population
2. σ known, large sample
3. σ unknown, normal population
4. σ unknown, large sample

For these scenarios, the procedure is derived on the basis of the distribution of the sample mean.

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Estimation of population mean: σ known

Covered scenarios:

I. The sample is large. [Rule of thumb: $n \geq 30$]

II. The population distribution of X is (approximately) normal.

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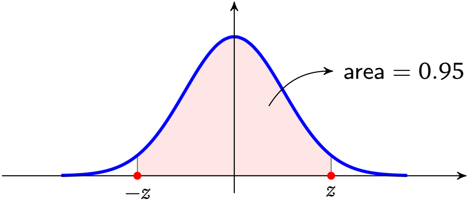
Confidence interval for μ

Assuming either I or II, the 95% confidence interval for μ is

$$\text{sample mean} \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

where z is the value indicated below.



Standard normal distribution

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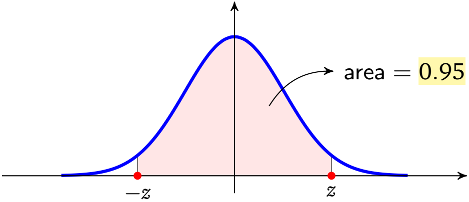

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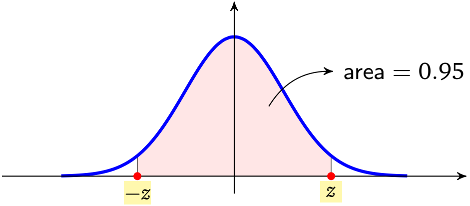

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&\approx \boxed{128.07 \pm 7.74\,\mathrm{Kg}} = (120.33, 135.81)
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