American University of Beirut

STAT 210: Elementary Statistics for Sciences

2022–2023 Fall

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Chapter 8

Estimation of mean and proportion

Part 2

Estimation of population parameters

Estimation of population parameters

Interval estimate/estimator

A 95% confidence interval for a population parameter θ is an interval of the form

$$\text{point estimate} \pm \text{margin of error}$$

(calculated on the basis of a random sample) that is meant to estimate the true value of θ.

The procedure used to obtain a confidence interval is called an interval estimator.

Estimation of population mean

Finding confidence intervals for the population mean

Suppose we want to estimate the population mean μ of a variable X based on a random sample of size n.

We will consider the following scenarios:

1. σ known, normal population
2. σ known, large sample
3. σ unknown, normal population
4. σ unknown, large sample

For these scenarios, the procedure is derived on the basis of the distribution of the sample mean.

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Estimation of population mean: σ known

Covered scenarios:

I. The sample is large. [Rule of thumb: $n \geq 30$]

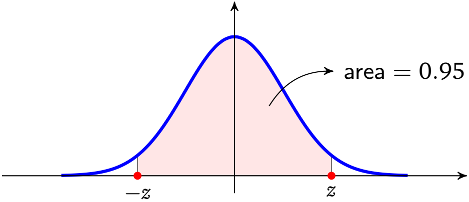
II. The population distribution of X is (approximately) normal.

Confidence interval for μ

Assuming either I or II, the 95% confidence interval for μ is

$$\text{sample mean} \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

where z is the value indicated below.



Standard normal distribution

Estimation of population mean: σ known

Example (Weight of gorillas)

The weights of a sample of 70 adult gorillas (in Kg):

w_1	w_2	w_3	w_4	$\cdots$	w_{70}	$\overline{w}$	s
150.81	92.25	92.00	97.98	$\cdots$	153.89	128.07	34.286

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Using the applet for normal distribution, $z \approx 1.95996$.

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Solving for z, we get $z \approx$.

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Solving for z, we get $z \approx 1.26621$.

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Using the applet for normal distribution, ? $\approx$.

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Using the applet for normal distribution, $? \approx \boxed{0.7946} \approx 79.5\%$.

Estimation of population mean: σ known

Example (Weight of gorillas)

The researchers decide to take a larger sample that allows them to estimate the population mean weight of the gorillas with

margin of error $\leq 5\,\text{Kg}$ confidence level $\geq 95\%$

As before, (by some miracle) they know $\sigma = 33.038\,\text{Kg}$.

Q3 How large must the sample be?

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Ideally, we would like

$$0.95 = \text{confidence level} = ?$$

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Given the confidence level $? = 0.95$, we can use the applet for normal distribution to find $z \approx 1.95996$.

Solving for n, we get $n \approx 167.72$.

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$$\text{margin of error} \leq 5\,\text{Kg} \qquad \text{confidence level} \geq 95\%$$

As before, (by some miracle) they know $\sigma = 33.038\,\text{Kg}$.

Q3 How large must the sample be?

A We know σ, and the sample is relatively large. Therefore,

$$?\%\text{-CI} = \text{sample mean} \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

Ideally, we would like

$$0.95 = \text{confidence level} = ?$$

$$5\,\text{Kg} = \text{margin of error} = z \cdot \frac{\sigma}{\sqrt{n}} = 1.95996 \cdot \frac{33.038}{\sqrt{n}}$$

Given the confidence level $? = 0.95$, we can use the applet for normal distribution to find $z \approx 1.95996$.

Solving for n, we get $n \approx 167.72$. [Warning: n must be a whole number!]

Estimation of population mean: σ known

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Thus, a sample of size $n \geq \boxed{168}$ would be sufficient.

Estimation of population mean

Finding confidence intervals for the population mean

Suppose we want to estimate the population mean μ of a variable X based on a random sample of size n.

We will consider the following scenarios:

1. σ known, normal population
2. σ known, large sample
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Covered scenarios:

I. The sample is large. [Rule of thumb: $n \geq 30$]

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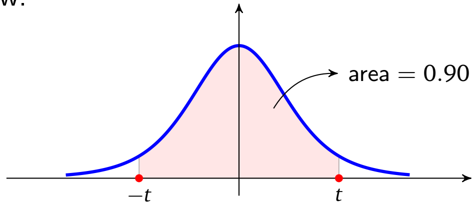

t-distribution with $df = n - 1$ degrees of freedom

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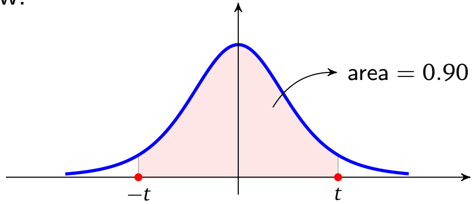

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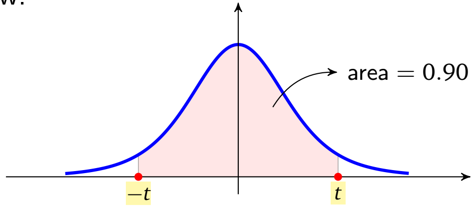

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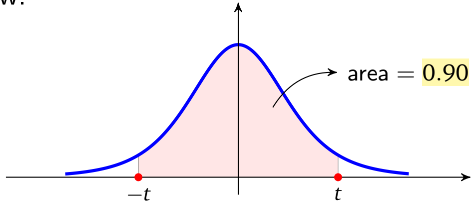
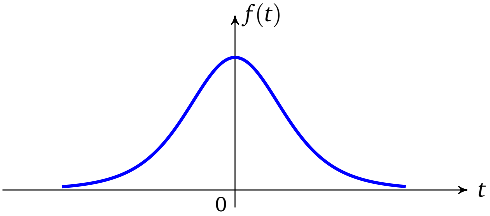


t-distribution with $df = n - 1$ degrees of freedom

Student's t-distribution

The t-distribution with parameter df (# of degrees of freedom) is another specific bell-shaped probability density function:



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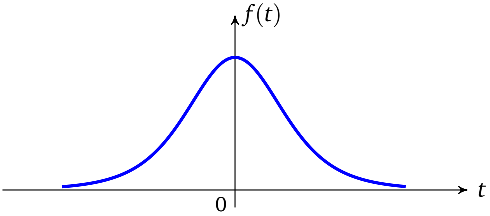

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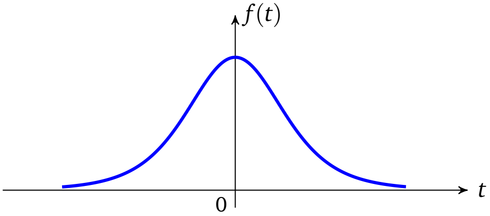

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The t-distribution is symmetric and unimodal. Its right and left tails extend indefinitely, which is to say:

The possible values of a t-RV are all numbers
$-\infty < t < +\infty.$

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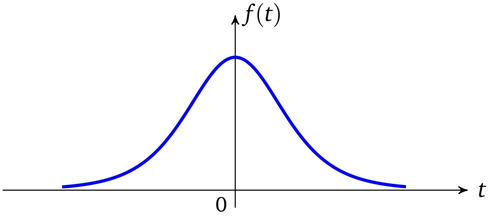

Remark 1

The mean of a t-distribution is 0, but its standard deviation is <u>somewhat larger</u> than 1.

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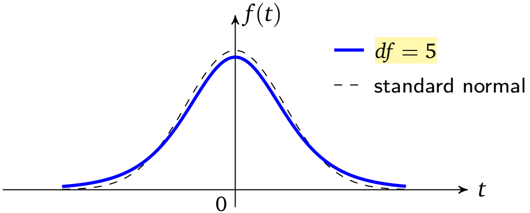

Remark 2

When df is small, the t-distribution deviates significantly from the standard normal distribution.

The larger df, the closer is the t-distribution to the standard normal distribution.

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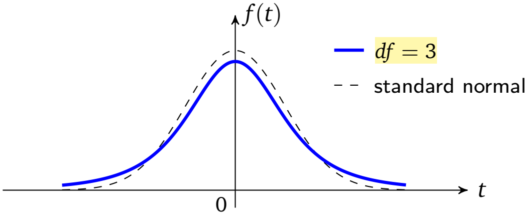

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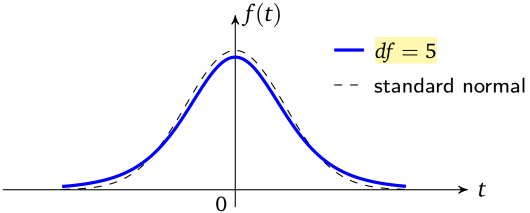

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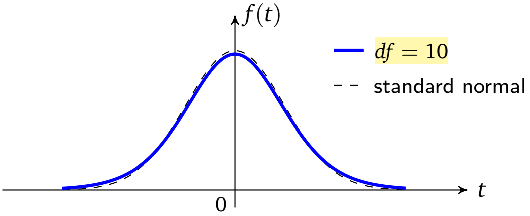

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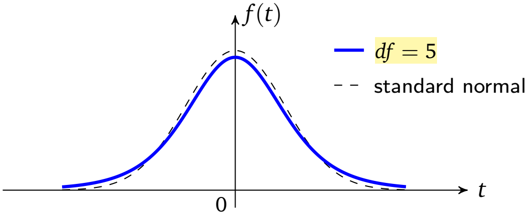

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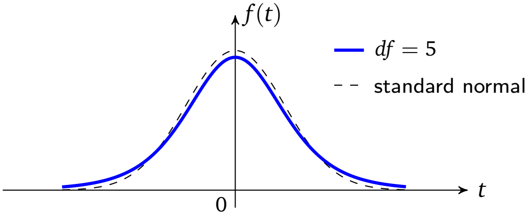

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Recall that if the sample is large or if the population is normally distributed, then the sample mean $\overline{X}$ is approximately normally distributed.

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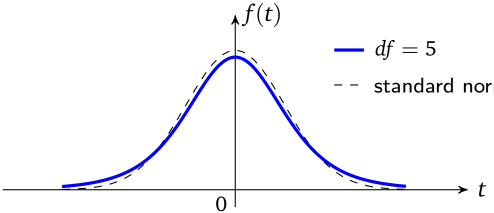

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Recall that if the sample is large or if the population is normally distributed, then the sample mean $\overline{X}$ is approximately normally distributed.
In particular, its standardized version

$$Z := \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

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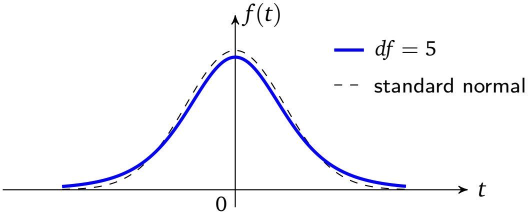

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Recall that if the sample is large or if the population is normally distributed, then the sample mean $\overline{X}$ is approximately normally distributed.
If we standardize $\overline{X}$ with the sample standard deviation S instead of the population standard deviation σ, then

$$T := \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

has the t-distribution with $df = n - 1$ degrees of freedom.

Estimation of population mean: σ unknown

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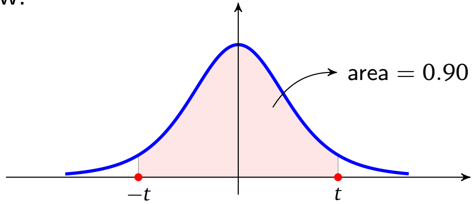

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Example (Weight of gorillas)

The weights of a sample of 70 adult gorillas (in Kg):

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Using the applet for t-distribution (with $df = n - 1 = 69$), we find $t \approx$.

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$$\begin{aligned} 95\%\text{-CI} &= \text{sample mean} \pm t \cdot \frac{s}{\sqrt{n}} \\ &\approx 128.07 \pm 1.99495 \cdot \frac{34.286}{\sqrt{70}} \\ &\approx \boxed{128.07 \pm 8.18\,\text{Kg}} = (119.89, 136.25) \end{aligned}$$

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Example (Weight of gorillas)

The weights of a sample of 70 adult gorillas (in Kg):

w_1	w_2	w_3	w_4	$\cdots$	w_{70}	$\overline{w}$	s
150.81	92.25	92.00	97.98	$\cdots$	153.89	128.07	34.286

~~Suppose (by some miracle) we know $\sigma = 33.038$ Kg.~~

Q2 What would be the level of confidence if we require the margin of error for estimating the population mean to be 5 Kg?

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