American University of Beirut

STAT 210: Elementary Statistics for Sciences

2022–2023 Fall

Siamak Taati

Chapter 8
Estimation of mean and proportion
Part 3

Estimation of population parameters

Estimation of population parameters

Interval estimate/estimator

A 95% confidence interval for a population parameter θ is an interval of the form

$$\text{point estimate} \pm \text{margin of error}$$

(calculated on the basis of a random sample) that is meant to estimate the true value of θ.

The procedure used to obtain a confidence interval is called an interval estimator.

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Finding confidence intervals for the population proportion

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II. Either the sample is taken with replacement,
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The procedure is derived on the basis of the (approximate) distribution of the sample proportion.

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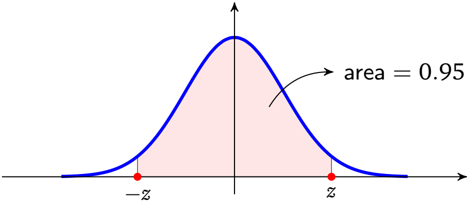
Confidence interval for p

Under conditions I and II, the 95% confidence interval for p is

$$\widehat{p} \pm z \cdot \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$$

where $\widehat{p}$ is the sample proportion and z is the value indicated below.



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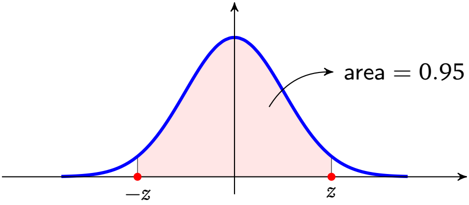

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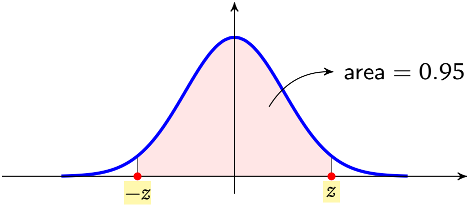

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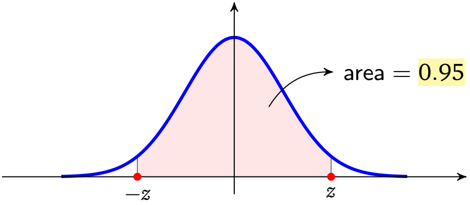

Standard normal distribution

Estimation of population proportion

Example (Gene with two alleles)

A certain type of gene in humans has two alleles (versions) A and B. Researchers would like to know the proportion p of individuals with allele A.

Since they do not have access to the entire population, they decide to take a random sample of individuals, identify their alleles, and use this data as evidence to estimate the population proportion p.

Estimation of population proportion

Example (Gene with two alleles)

The researchers take a sample of 50 individuals and identify their alleles of this gene:

1	2	3	4	···	50	# of As	# of Bs
B	B	A	A	···	B	31	19

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$$\widehat{p} = \frac{31}{50} = \boxed{0.62} = 62\%$$

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&\approx \boxed{0.62 \pm 0.135} = (0.485, 0.755)
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Solving for z, we get $z \approx 0.7283957$.

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Using the applet for normal distribution, $? \approx \boxed{0.5336} \approx 53.4\%$.

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The researchers decide to take a larger sample that allows them to estimate the population proportion of allele A with

margin of error ≤ 0.05 confidence level $\geq 95\%$

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The researchers decide to take a larger sample that allows them to estimate the population proportion of allele A with

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Thus, a sample of size $n \geq \boxed{385}$ would be sufficient.

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Thus, a sample of size $n \geq \boxed{363}$ would be sufficient.

Estimation of population parameters

Interval estimate/estimator

A 95% confidence interval for a population parameter θ is an interval of the form

$$\text{point estimate} \pm \text{margin of error}$$

(calculated on the basis of a random sample) that is meant to estimate the true value of θ.

The procedure used to obtain a confidence interval is called an interval estimator.

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The confidence level of 95% is a property of the interval estimator (i.e., the procedure).

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Consider the following experiment:

> Take a random sample of size n and use the interval estimator to form a confidence interval for θ.

If we repeat this experiment many many times (each time with a fresh sample, independently of the previous ones), then

- 95% of the times, the formed confidence interval will contain the true value of θ, and
- the remaining 5% of the times it misfires.