American University of Beirut

STAT 210: Elementary Statistics for Sciences

2022–2023 Fall

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Chapter 9

Testing hypotheses about mean and proportion

Part 3

Statistical hypothesis testing

Statistical hypothesis testing

Competing hypotheses

H_0 The null hypothesis (the default model)

H_1 The alternative hypothesis (the alternative claim)

Objective

Decide whether we should abandon H_0 in favor of H_1.

Evidence

A random sample

Strategy

We use a statistic T and compare [e.g., sample mean]

1. The observed value of T
2. The distribution of T as suggested by H_0,

We ask ourselves:

Q Is the observed value of T too extreme for H_0 to be plausible?

Statistical hypothesis testing

Competing hypotheses

H_0 The null hypothesis (the default model)

H_1 The alternative hypothesis (the alternative claim)

Objective

Decide whether we should abandon H_0 in favor of H_1.

Possible outcomes

- Reject H_0
  (Meaning: The evidence is statistically significant in favor of H_1.)
- Fail to reject H_0
  (Meaning: The evidence is inconclusive.)

Remark

The statistical evidence is always meant to be against H_0.
Even if the evidence is weak, it can never lead us to accept H_0.

Statistical hypothesis testing

Competing hypotheses

H_0 The null hypothesis (the default model)

H_1 The alternative hypothesis (the alternative claim)

Remark

Since the evidence is limited, our judgment can be erroneous!

Two types of errors

- Type I (false rejection):

  H_0 is true but we incorrectly reject it.

- Type II (failure to reject):

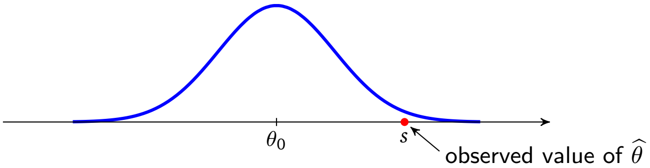
  H_0 is not true but we fail to reject it.

Tests about a population parameter

Competing hypotheses	Competing hypotheses	Competing hypotheses
H_0 $\theta = \theta_0$	H_0 $\theta = \theta_0$	H_0 $\theta = \theta_0$
H_1 $\theta < \theta_0$	H_1 $\theta \neq \theta_0$	H_1 $\theta > \theta_0$
left-tailed test	two-tailed test	right-tailed test

Test statistic

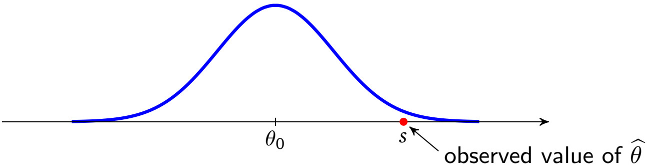
A point estimator $\widehat{\theta}$ for θ. [e.g., sample mean $\widehat{\mu} = \overline{X}$]



Distribution of $\widehat{\theta}$ assuming H_0

Q Is the observed value $\widehat{\theta} = s$ too extreme for H_0 to be plausible?

Tests about a population parameter



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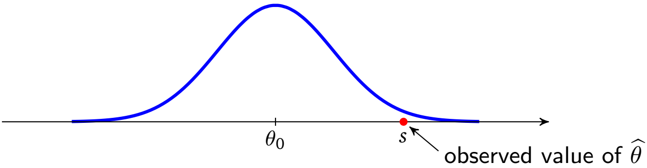

Distribution of $\widehat{\theta}$ assuming H_0

Q Is the observed value $\widehat{\theta} = s$ too extreme for H_0 to be plausible?

In order to make a fair and rigorous judgment:

We decide on a significance level α before observing the evidence, and choose the criterion for judgment in such a way that the probability of Type I error is α.

Tests about a population parameter

Significance level

The probability of Type I error

$$\alpha := \mathbb{P}(\mathsf{H}_0 \text{ is rejected} \mid \mathsf{H}_0 \text{ is true})$$

is called the significance level.

Tests about a population parameter

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Q How can we choose the criterion for judgment based on the significance level α?

Tests about a population parameter

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Q How can we choose the criterion for judgment based on the significance level α?

A1 Identify the rejection region for H_0 by computing its critical values.

Tests about a population parameter

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Q How can we choose the criterion for judgment based on the significance level α?

A1 Identify the rejection region for H_0 by computing its critical values. Reject H_0 if and only if the observed value falls within the rejection region.

Tests about a population parameter

Significance level

The probability of Type I error

$$\alpha := \mathbb{P}(\text{H}_0 \text{ is rejected} \mid \text{H}_0 \text{ is true})$$

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Q How can we choose the criterion for judgment based on the significance level α?

A1 Identify the rejection region for H_0 by computing its critical values. Reject H_0 if and only if the observed value falls within the rejection region.

A2 Compute the probability that, assuming H_0, the test statistic $\widehat{\theta}$ takes a value which is at least as extreme as the observed value (the p-value).

Tests about a population parameter

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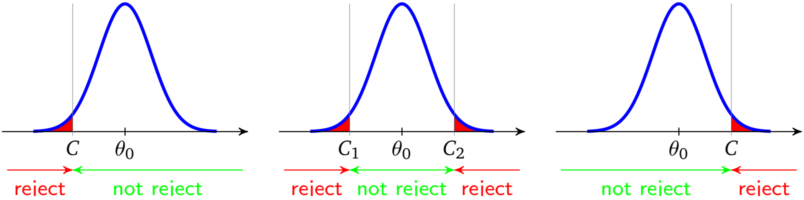
A2 Compute the probability that, assuming H_0, the test statistic $\widehat{\theta}$ takes a value which is at least as extreme as the observed value (the p-value). Reject H_0 if and only if this probability is at most α.

Tests about a population parameter



H_1: $\theta < \theta_0$ $\quad$ H_1: $\theta \neq \theta_0$ $\quad$ H_1: $\theta > \theta_0$

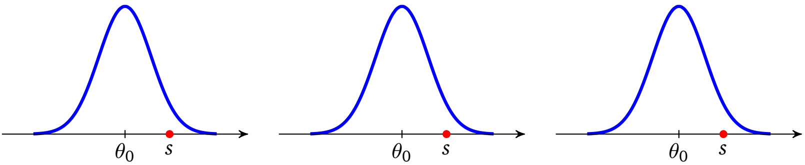
Q How can we choose the criterion for judgment based on the significance level α?

Rejection region approach

Depending on the type of alternative hypothesis, choose the critical values in such a way that the red area (= the probability of Type I error) is α.

Reject H_0 if and only if the observed value falls within the rejection region.

Tests about a population parameter



H$_1$: $\theta < \theta_0$ H$_1$: $\theta \neq \theta_0$ H$_1$: $\theta > \theta_0$

Q How can we choose the criterion for judgment based on the significance level α?

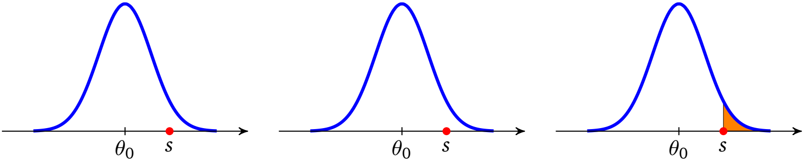
p-value approach

Depending on the type of alternative hypothesis, compute

$$\text{p-value} := \mathbb{P}(\widehat{\theta} \text{ is at least as extreme as } s \mid \text{H}_0 \text{ is true})$$

Reject H$_0$ <u>if and only if</u> p-value $\leq \alpha$.

Tests about a population parameter



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p-value approach

Depending on the type of alternative hypothesis, compute

$$\begin{aligned}\text{p-value} &:= \mathbb{P}(\widehat{\theta} \text{ is at least as extreme as } s \mid \text{H}_0 \text{ is true}) \\ &= \langle \text{the orange area} \rangle\end{aligned}$$

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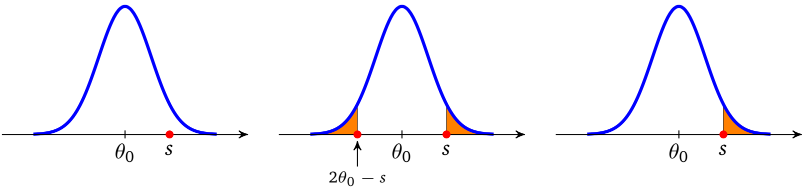
Tests about a population parameter



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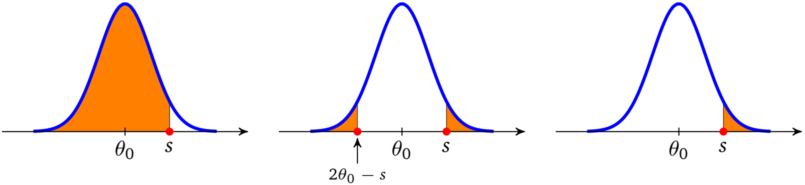

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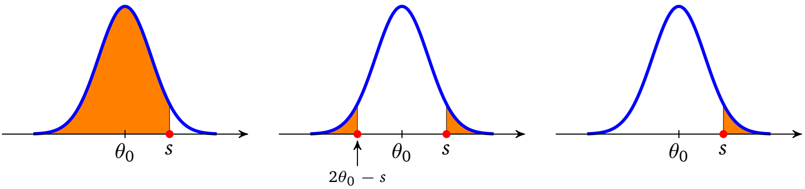
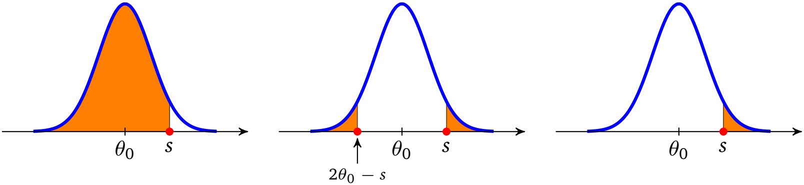


H$_1$: $\theta < \theta_0$ H$_1$: $\theta \neq \theta_0$ H$_1$: $\theta > \theta_0$

Remark

The two approaches are logically equivalent.

Tests about a population parameter



H$_1$: $\theta < \theta_0$ H$_1$: $\theta \neq \theta_0$ H$_1$: $\theta > \theta_0$

Remark

The two approaches are logically equivalent.

Nevertheless, the p-value approach has an advantage:

> The p-value provides a standardized measure of the strength (or statistical significance) of the evidence against the null hypothesis: The smaller the p-value, the stronger the evidence.

Tests about a population parameter

We now focus on the cases in which the parameter of interest is either the population mean or the population proportion.

Tests about the population mean

We consider the following scenarios:

1. σ known, normal population
2. σ known, large sample
3. σ unknown, normal population
4. σ unknown, large sample

Tests about the population proportion

We consider the following scenarios:

1. large sample with replacement
2. large sample, much larger population

Tests about a population parameter

We now focus on the cases in which the parameter of interest is either the population mean or the population proportion.

Tests about the population mean

We consider the following scenarios:

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3. σ unknown, normal population
4. σ unknown, large sample

Tests about the population proportion

We consider the following scenarios:

1. large sample with replacement
2. large sample, much larger population

Tests about a population mean: σ known

Example (Fish and pollution)

The fish from a certain species naturally contains a chemical that, in large quantities, is harmful to humans. The concentration of the chemical in healthy fish is normally distributed with mean 0.167 ppm and standard deviation 0.044 ppm.

Tests about a population mean: σ known

Example (Fish and pollution)

The fish from a certain species naturally contains a chemical that, in large quantities, is harmful to humans. The concentration of the chemical in healthy fish is normally distributed with mean 0.167 ppm and standard deviation 0.044 ppm.

Researchers at AUB would suspect that due to pollution, the concentration of the chemical in the fish captured from the coast of Beirut is higher than usual.

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Q What are the competing hypotheses?

A H_0 $\mu = 0.167\,\text{ppm}$

H_1 $\mu > 0.167\,\text{ppm}$

where μ is the population mean concentration of the chemical in the fish captured from the coast of Beirut.

Tests about a population mean: σ known

Example (Fish and pollution)

Concentration in healthy fish: $\boxed{\text{normal with } \mu_0 = 0.167\,\text{ppm and } \sigma_0 = 0.044\,\text{ppm}}$

Concentration in fish from Beirut: $\boxed{\text{H}_0\text{: } \mu = 0.167\,\text{ppm} \qquad \text{H}_1\text{: } \mu > 0.167\,\text{ppm}}$

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To test their hypothesis, the researchers collect a random sample of 10 fish from the coast of Beirut and measure the concentration of the chemical in them.

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Q What is a suitable statistic for this test?

A The sample mean $\overline{X}$ of the concentration of the chemical.

Tests about a population mean: σ known

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To test their hypothesis, the researchers collect a random sample of 10 fish from the coast of Beirut and measure the concentration of the chemical in them.

Q What is a suitable statistic for this test?

A The sample mean $\overline{X}$ of the concentration of the chemical.

Q What is the distribution of $\overline{X}$ assuming H_0?

A Normal with mean $\mu_0 = 0.167\,\text{ppm}$
and standard deviation $\frac{\sigma_0}{\sqrt{n}} = \frac{0.044}{\sqrt{10}} \approx 0.013914\,\text{ppm}.$

Tests about a population mean: σ known

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The average concentration of the chemical among the 10 sampled fish turns out to be $0.203\,\text{ppm}$.

Tests about a population mean: σ known

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Q At significance level $\alpha = 5\%$, how should the researchers conclude?

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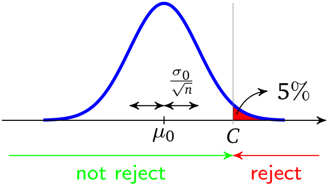
The average concentration of the chemical among the 10 sampled fish turns out to be 0.203 ppm.

Q At significance level $\alpha = 5\%$, how should the researchers conclude?

A1 The critical value C is such that

$$\mathbb{P}(\bar{X} \geq C \mid H_0) = \alpha$$



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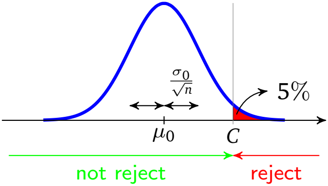
A1 The critical value C is such that

$$\mathbb{P}(\overline{X} \geq C \mid \text{H}_0) = \alpha$$

Using the applet for normal distribution, we find $C \approx$.



Tests about a population mean: σ known

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The average concentration of the chemical among the 10 sampled fish turns out to be 0.203 ppm.

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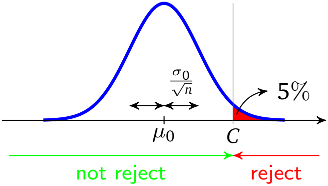
A1 The critical value C is such that

$$\mathbb{P}(\overline{X} \geq C \mid H_0) = \alpha$$

Using the applet for normal distribution, we find $C \approx 0.18989$.



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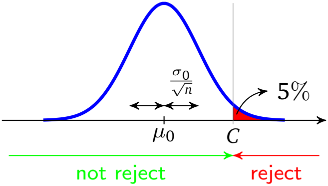
A1 The critical value C is such that

$$\mathbb{P}(\overline{X} \geq C \mid \text{H}_0) = \alpha$$

Using the applet for normal distribution, we find $C \approx 0.18989$.

Since $0.203 \geq C$, the researchers must reject the null hypothesis.



Tests about a population mean: σ known

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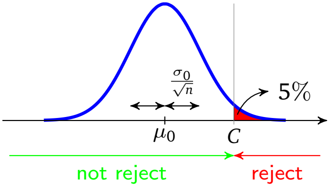

Conclusion: The evidence is statistically significant in support of the claim that the concentration of the chemical is higher in fish from Beirut.

Tests about a population mean: σ known

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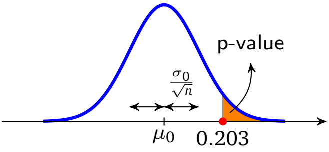
The average concentration of the chemical among the 10 sampled fish turns out to be $0.203\,$ppm.

Q At significance level $\alpha = 5\%$, how should the researchers conclude?

A2 Using the applet for normal distribution, we can find the p-value as

$$\text{p-value} = \mathbb{P}(\overline{X} \geq 0.203 \mid \text{H}_0)$$

$$\approx$$



Tests about a population mean: σ known

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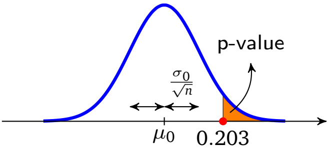
A2 Using the applet for normal distribution, we can find the p-value as

$$\text{p-value} = \mathbb{P}(\overline{X} \geq 0.203 \mid H_0) \approx \boxed{0.00484}\,.$$



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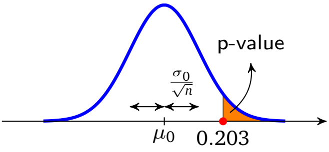
Since $0.00484 \leq \alpha$, the researchers should reject the null hypothesis.



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Example (Fish and pollution)

Concentration in healthy fish: $\boxed{\text{normal with } \mu_0 = 0.167\,\text{ppm and } \sigma_0 = 0.044\,\text{ppm}}$

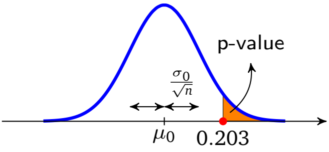
Concentration in fish from Beirut: $\boxed{\text{H}_0\text{: } \mu = 0.167\,\text{ppm} \qquad \text{H}_1\text{: } \mu > 0.167\,\text{ppm}}$

The average concentration of the chemical among the 10 sampled fish turns out to be 0.203 ppm.

Q At significance level $\alpha = 5\%$, how should the researchers conclude?

$\boxed{\text{A2}}$ Using the applet for normal distribution, we can find the p-value as

$$\text{p-value} = \mathbb{P}(\overline{X} \geq 0.203 \mid \text{H}_0) \approx \boxed{0.00484}\,.$$



Since $0.00484 \leq \alpha$, the researchers should reject the null hypothesis.

Conclusion: The evidence is statistically significant (p-value $< 0.5\%$) in support of the claim that the concentration of the chemical is higher in fish from Beirut.

Tests about a population mean: σ known

Covered scenarios:

I. The sample is large. [Rule of thumb: $n \geq 30$]

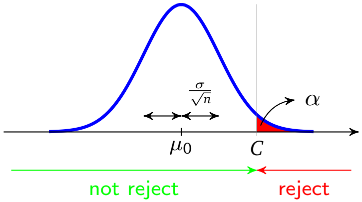
II. The population distribution of X is (approximately) normal.

Tests about μ (rejection region approach)

$$\boxed{H_0: \ \mu = \mu_0 \qquad H_1: \ \mu > \mu_0}$$

We can use the sample mean $\overline{X}$ as the test statistic.

Assuming either I or II, at significance level α, we reject H_0 if $\overline{X} \geq C$, where C is the critical value indicated below.



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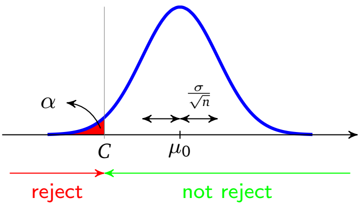

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$$\boxed{H_0: \ \mu = \mu_0 \qquad H_1: \ \mu \neq \mu_0}$$

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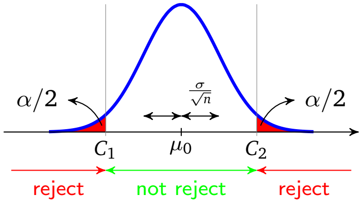
Assuming either I or II, at significance level α, we reject H_0 if $\overline{X} \leq C_1$ or $\overline{X} \geq C_2$, where C_1 and C_2 are as indicated below.



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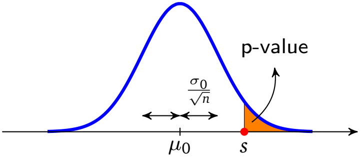
Assuming either I or II, at significance level α,

$$\text{p-value} = \mathbb{P}(\overline{X} \geq s \mid \text{H}_0)$$

where s is the observed value of the sample mean.

We reject H$_0$ if p-value $\leq \alpha$.



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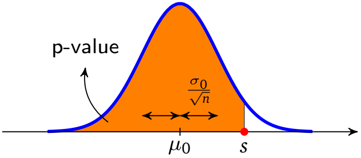

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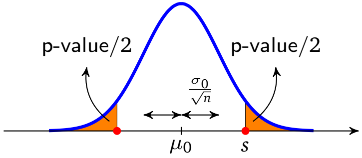
We can use the sample mean $\overline{X}$ as the test statistic.

Assuming either I or II, at significance level α,

$$\text{p-value} = 2\,\mathbb{P}(\overline{X} \geq s \mid H_0) \qquad \text{if } s > \mu_0$$

where s is the observed value of the sample mean.

We reject H_0 if p-value $\leq \alpha$.



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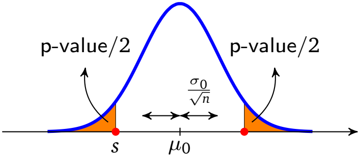

Tests about a population parameter

We now focus on the cases in which the parameter of interest is either the population mean or the population proportion.

Tests about the population mean

We consider the following scenarios:

1. σ known, normal population
2. σ known, large sample
3. σ unknown, normal population
4. σ unknown, large sample

Tests about the population proportion

We consider the following scenarios:

1. large sample with replacement
2. large sample, much larger population

Tests about a population parameter

We now focus on the cases in which the parameter of interest is either the population mean or the population proportion.

Tests about the population mean

We consider the following scenarios:

1. σ known, normal population
2. σ known, large sample
3. σ unknown, normal population
4. σ unknown, large sample

Tests about the population proportion

We consider the following scenarios:

1. large sample with replacement
2. large sample, much larger population

Tests about a population mean: σ unknown

Covered scenarios:

I. The sample is large. [Rule of thumb: $n \geq 30$]

II. The population distribution of X is (approximately) normal.

Tests about μ (rejection region approach)

$$\boxed{\mathrm{H}_0\text{: } \mu = \mu_0 \qquad \mathrm{H}_1\text{: } \mu > \mu_0}$$

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$$\boxed{\mathsf{H}_0\text{: } \mu = \mu_0 \qquad \mathsf{H}_1\text{: } \mu > \mu_0}$$

Choice of statistic

Tests about a population mean: σ unknown

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Tests about μ (rejection region approach)

$$\boxed{\text{H}_0:\ \mu = \mu_0 \qquad \text{H}_1:\ \mu > \mu_0}$$

Choice of statistic

When σ is unknown, we cannot use the sample mean $\overline{X}$ as the test statistic.

Indeed, assuming H_0, the sample mean $\overline{X}$ is still (approximately) normally distributed, but we do not have access to its standard deviation $\sigma/\sqrt{n}$.

Tests about a population mean: σ unknown

Covered scenarios:

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Tests about μ (rejection region approach)

$$\boxed{\mathrm{H}_0\text{: } \mu = \mu_0 \qquad \mathrm{H}_1\text{: } \mu > \mu_0}$$

Choice of statistic

Instead, recall that (assuming H_0)

$$T := \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$$

has the t-distribution with $df = n - 1$.

Tests about a population mean: σ unknown

Covered scenarios:

I. The sample is large. [Rule of thumb: $n \geq 30$]

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Choice of statistic

Instead, recall that (assuming H$_0$)

$$T := \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$$

has the t-distribution with $df = n - 1$.

We can use T as the test statistic.
(For a right-tailed test, $T \gg 0$ would be evidence against H$_0$.)

Tests about a population mean: σ unknown

Covered scenarios:

I. The sample is large. [Rule of thumb: $n \geq 30$]

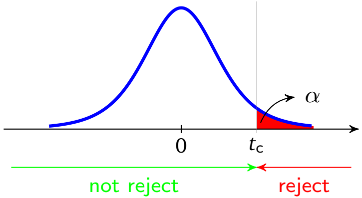
II. The population distribution of X is (approximately) normal.

Tests about μ (rejection region approach)

$$\boxed{\mathrm{H}_0: \ \mu = \mu_0 \qquad \mathrm{H}_1: \ \mu > \mu_0}$$

We can use $T := \frac{\overline{X}-\mu_0}{S/\sqrt{n}}$ as the test statistic. [S: sample standard deviation]

Assuming either I or II, at significance level α, we reject H_0 if $T \geq t_c$, where t_c is the critical value indicated below.



t-distribution with $df = n - 1$

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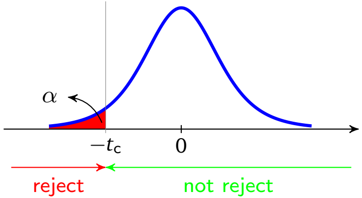
$$\boxed{\mathrm{H}_0\text{: } \mu = \mu_0 \qquad \mathrm{H}_1\text{: } \mu < \mu_0}$$

We can use $T := \frac{\overline{X}-\mu_0}{S/\sqrt{n}}$ as the test statistic. [S: sample standard deviation]

Assuming either I or II, at significance level α, we reject H_0 if $T \leq -t_{\mathsf{c}}$, where $-t_{\mathsf{c}}$ is the critical value indicated below.



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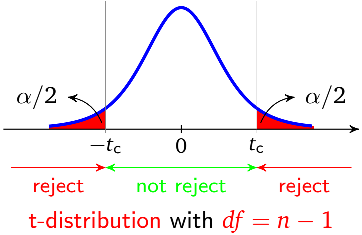
Assuming either I or II, at significance level α, we reject H$_0$ if $T \geq t_\mathsf{c}$ or $T \leq -t_\mathsf{c}$, where C is the critical value indicated below.



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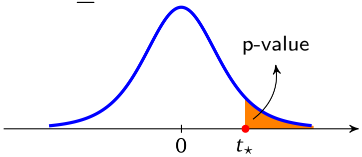
Assuming either I or II, at significance level α,

$$\text{p-value} = \mathbb{P}(T \geq t_\star \mid \text{H}_0)$$

where $t_\star$ is the observed value of the test statistic T.

We reject H_0 if p-value $\leq \alpha$.



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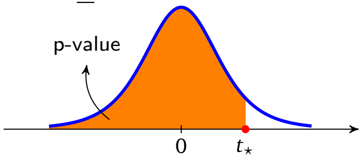

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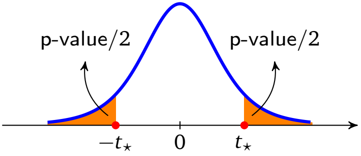
We can use $T := \frac{\overline{X}-\mu_0}{S/\sqrt{n}}$ as the test statistic. [S: sample standard deviation]

Assuming either I or II, at significance level α,

$$\text{p-value} = 2\,\mathbb{P}(T \geq t_\star \mid \mathrm{H}_0) \qquad \text{if } t_\star > 0$$

where $t_\star$ is the observed value of the test statistic T.

We reject H_0 if p-value $\leq \alpha$.



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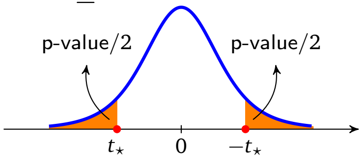
where $t_\star$ is the observed value of the test statistic T.

We reject H_0 if p-value $\leq \alpha$.



t-distribution with $df = n - 1$

Tests about a population mean: σ unknown

Example (Quality control)

A dairy factory produces labneh. According to the specification, one jar of labneh produced by the factory must on average weigh 375 g. It can be assumed that the weight of one jar is normally distributed.

Tests about a population mean: σ unknown

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The quality control specialists at the factory would like to verify this claim.

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Q What are the competing hypotheses?

Tests about a population mean: σ unknown

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A dairy factory produces labneh. According to the specification, one jar of labneh produced by the factory must on average weigh 375 g. It can be assumed that the weight of one jar is normally distributed.

The quality control specialists at the factory would like to verify this claim.

Q What are the competing hypotheses?

A H$_0$ $\mu = 375\,\mathrm{g}$

H$_1$ $\mu \neq 375\,\mathrm{g}$

where μ is the actual average weight of a jar of labneh.

Tests about a population mean: σ unknown

Example (Quality control)

Specified weight of a jar of labneh: $\boxed{\text{normal with } \mu_0 = 375\,\text{g and } \sigma_0 = ?}$

Actual weight of a jar of labneh: $\boxed{\text{H}_0\text{: } \mu = 375\,\text{g} \qquad \text{H}_1\text{: } \mu \neq 375\,\text{g}}$

Tests about a population mean: σ unknown

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Specified weight of a jar of labneh: $\boxed{\text{normal with } \mu_0 = 375\,\text{g and } \sigma_0 = ?}$

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To test the claim, the quality control specialists collect a random sample of 10 jars of labneh from the factory and measure their weight.

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To test the claim, the quality control specialists collect a random sample of 10 jars of labneh from the factory and measure their weight.

Q What is a suitable statistic for this test?

A The mean weight $\overline{W}$ of the sampled jars is normally distributed with mean 375 g, but we do not have access to its standard deviation $\sigma_0/\sqrt{10}$.

Tests about a population mean: σ unknown

Example (Quality control)

Specified weight of a jar of labneh: $\boxed{\text{normal with } \mu_0 = 375\,\text{g and } \sigma_0 = ?}$

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To test the claim, the quality control specialists collect a random sample of 10 jars of labneh from the factory and measure their weight.

Q What is a suitable statistic for this test?

A Instead, we use the standardized mean weight

$$T := \frac{\overline{W} - 375}{S/\sqrt{10}}$$

where S is the sample standard deviation.

Tests about a population mean: σ unknown

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A t distribution with $df = 10 - 1 = 9$.

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From the collected sample of 10, the sample mean turns out to be $\overline{w} = 371.8\,\text{g}$ and the sample standard deviation $s = 5.24\,\text{g}$.

Tests about a population mean: σ unknown

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Q What is the observed value of the test statistic?

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From the collected sample of 10, the sample mean turns out to be $\overline{w} = 371.8\,\text{g}$ and the sample standard deviation $s = 5.24\,\text{g}$.

Q What is the observed value of the test statistic?

A

$$t_\star = \frac{\overline{w} - \mu_0}{s/\sqrt{n}}$$

Tests about a population mean: σ unknown

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Specified weight of a jar of labneh: $\boxed{\text{normal with } \mu_0 = 375\,\text{g and } \sigma_0 = ?}$

Actual weight of a jar of labneh: $\boxed{H_0\!: \ \mu = 375\,\text{g} \qquad H_1\!: \ \mu \neq 375\,\text{g}}$

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Q What is the observed value of the test statistic?

$\boxed{\text{A}}$

$$t_\star = \frac{\overline{w} - \mu_0}{s/\sqrt{n}} = \frac{371.8 - 375}{5.24/\sqrt{10}}$$

Tests about a population mean: σ unknown

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Specified weight of a jar of labneh: $\boxed{\text{normal with } \mu_0 = 375\,\text{g and } \sigma_0 = ?}$

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Q What is the observed value of the test statistic?

A

$$t_\star = \frac{\overline{w} - \mu_0}{s/\sqrt{n}} = \frac{371.8 - 375}{5.24/\sqrt{10}} \approx \boxed{-1.931162}$$

Tests about a population mean: σ unknown

Example (Quality control)

Specified weight of a jar of labneh: $\boxed{\text{normal with } \mu_0 = 375\,\text{g and } \sigma_0 = ?}$

Actual weight of a jar of labneh: $\boxed{\text{H}_0\text{: } \mu = 375\,\text{g} \qquad \text{H}_1\text{: } \mu \neq 375\,\text{g}}$

From the collected sample of 10, the sample mean turns out to be $\overline{w} = 371.8\,\text{g}$ and the sample standard deviation $s = 5.24\,\text{g}$.

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A1 The critical value t_c is such that

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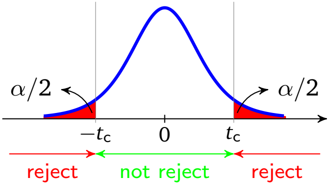

t-distribution with $df = n - 1$

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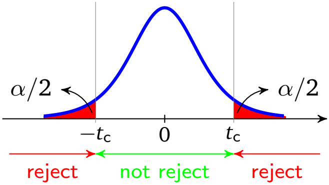
Using the applet for t distribution, we find $t_c \approx$.



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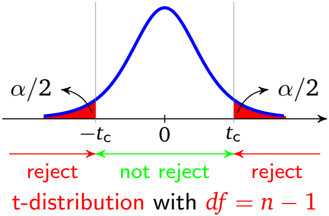
Using the applet for t distribution, we find $t_c \approx 2.26216$.



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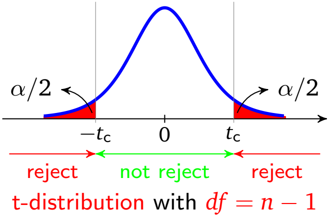
Since $-1.931162 \geq -t_c$, the quality control specialists cannot reject the null hypothesis.



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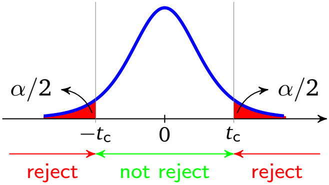

t-distribution with $df = n - 1$

Conclusion: The evidence against the validity of the product specification is not statistically significant.

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A2 Note that $t_\star = -1.931162 < 0$.
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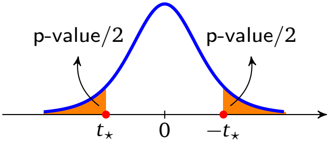
$$\text{p-value} = 2\,\mathbb{P}(T \leq -1.931162 \mid H_0)$$

$$\approx$$



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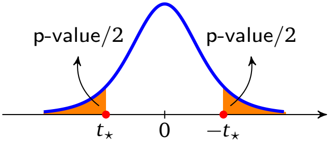

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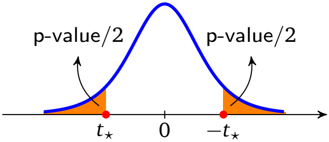

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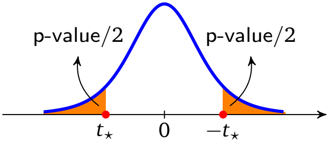

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Since $0.08552 > \alpha$, the quality control specialists cannot reject the null hypothesis.

Conclusion: The evidence against the validity of the product specification is not statistically significant.

Tests about a population parameter

We now focus on the cases in which the parameter of interest is either the population mean or the population proportion.

Tests about the population mean

We consider the following scenarios:

1. σ known, normal population
2. σ known, large sample
3. σ unknown, normal population
4. σ unknown, large sample

Tests about the population proportion

We consider the following scenarios:

1. large sample with replacement
2. large sample, much larger population

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Tests about a population proportion

Covered scenarios:

I. The sample is large and with replacement

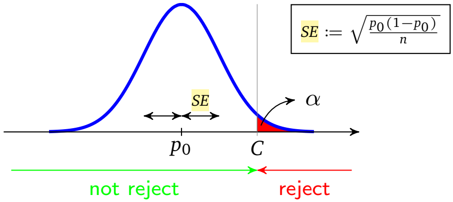
II. The sample is large, the population larger

Tests about p (rejection region approach)

$$\boxed{\mathrm{H}_0:\ p = p_0 \qquad \mathrm{H}_1:\ p > p_0}$$

We can use the sample proportion $\widehat{p}$ as the test statistic.

Assuming either I or II, at significance level α, we reject H_0 if $\widehat{p} \geq C$, where C is the critical value indicated below.



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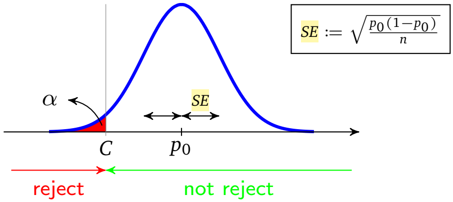

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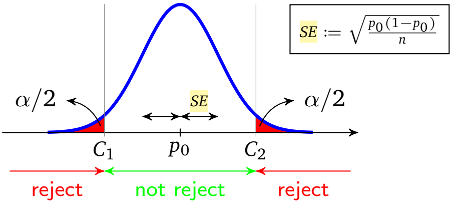
Assuming either I or II, at significance level α, we reject H_0 if $\widehat{p} \leq C_1$ or $\widehat{p} \geq C_2$, where C_1 and C_2 are as indicated below.



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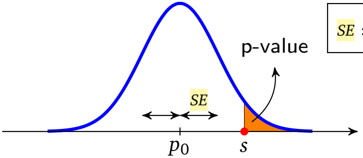
where s is the observed value of the sample proportion.

We reject H_0 if p-value $\leq \alpha$.



$$\boxed{SE := \sqrt{\frac{p_0(1-p_0)}{n}}}$$

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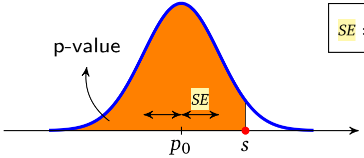

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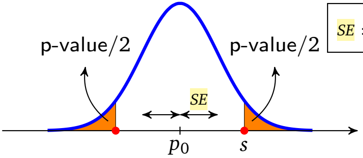

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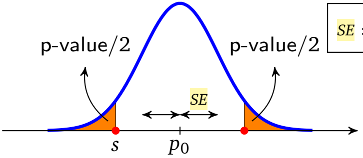

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