

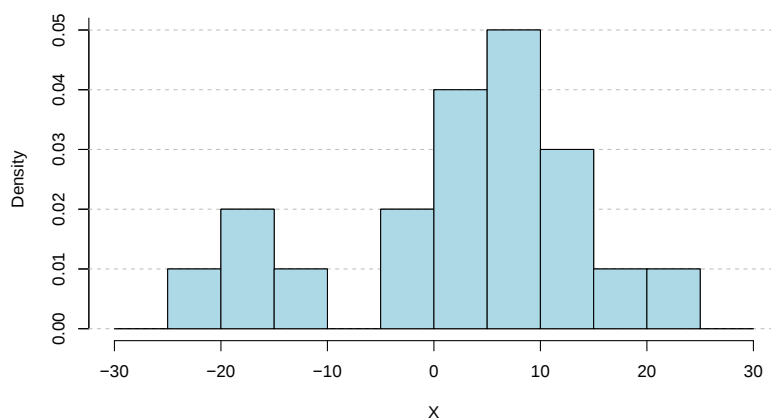
Full Name: _____

Student No: _____

Grade:

For each answer, **provide a short argument.**

1. (3 points) The following diagram is the histogram of a numerical variable X from a data set.



- (a) What percentage of values are between -10 and 10 ?

Answer: 55%

Solution: The area of the rectangles above the interval between -10 and 10 is $11/20 = 55\%$ of the total area.

- (b) What is the approximate value of the median of X ?

Answer: 5

Solution: The area of the rectangles on the left of 5 is the same as the area of the rectangles on the right of 5.

- (c) Does 12 lie in the upper quartile?

Answer: Yes

Solution: The area of the rectangles on the right of 10 is $1/4$ of the total area.

2. (4 points) A jar contains 3 blue balls, 2 red balls and 1 yellow ball. At random, we draw two balls from the jar, one after another, without replacement.

(a) What is an appropriate sample space for this experiment?

Solution: There are at least three options:

- Model 1: $\Omega = \{BB, BR, BY, RB, RR, RY, YB, YY\}$ (8 possible outcomes).
This model keeps track of the order, but does not distinguish between the balls that have the same color.
- Model 2: Number the 6 balls $1, 2, \dots, 6$, where the first three are blue, the next two are red and the last is yellow.
 $\Omega = \{(1, 2), (1, 3), (1, 4), \dots, (6, 5)\}$ ($6 \times 5 = 30$ possible outcomes).
This model keeps track of the order and distinguishes between balls of same color.
- Model 3: $\Omega = \{(2, 0, 0), (1, 1, 0), (1, 0, 1), (0, 2, 0), (0, 1, 1)\}$ (5 possible outcomes), where $(2, 0, 0)$ represents 2 blue balls, $(1, 1, 0)$ represents 1 blue ball and 1 red ball, and so on.
This models neither keeps track of the order, nor distinguishes between the balls of same color.

We will proceed with Model 1.

(b) Identify the probability of each outcome in the sample space.

Solution:

$\mathbb{P}(BB) = 3/6 \times 2/5 = 1/5$	$\mathbb{P}(BR) = 3/6 \times 2/5 = 1/5$	$\mathbb{P}(BY) = 3/6 \times 1/5 = 1/10$
$\mathbb{P}(RB) = 2/6 \times 3/5 = 1/5$	$\mathbb{P}(RR) = 2/6 \times 1/5 = 1/15$	$\mathbb{P}(RY) = 2/6 \times 1/5 = 1/15$
$\mathbb{P}(YB) = 1/6 \times 3/5 = 1/10$	$\mathbb{P}(YR) = 1/6 \times 2/5 = 1/15$	

(c) What is the probability that the two drawn balls have different colors?

Answer: $11/15 \approx 0.7333$

Solution:

$$\begin{aligned}
 \mathbb{P}(\langle \text{two balls with different colors} \rangle) &= \mathbb{P}(\{BR, BY, RB, RY, YB, YR\}) \\
 &= \mathbb{P}(BR) + \mathbb{P}(BY) + \mathbb{P}(RB) + \mathbb{P}(RY) + \mathbb{P}(YB) + \mathbb{P}(YR) \\
 &= \frac{1}{5} + \frac{1}{10} + \frac{1}{5} + \frac{1}{15} + \frac{1}{10} + \frac{1}{15} = \frac{11}{15}.
 \end{aligned}$$

Alternatively,

$$\begin{aligned}
 \mathbb{P}(\langle \text{two balls with different colors} \rangle) &= 1 - \mathbb{P}(\langle \text{two balls with the same color} \rangle) \\
 &= 1 - \mathbb{P}(\{BB, RR\}) \\
 &= 1 - \mathbb{P}(BB) - \mathbb{P}(RR) \\
 &= 1 - \frac{1}{5} - \frac{1}{15} = \frac{11}{15}.
 \end{aligned}$$

(d) If we know that the two drawn balls have the same color, what is the chance that they are blue?

Answer: $\frac{3}{4} = 0.75$

Solution: Let

$$E := \langle \text{both drawn balls are blue} \rangle = \{BB\} ,$$

$$C := \langle \text{drawn balls have same color} \rangle = \{BB, RR\} .$$

We have

$$\begin{aligned} & \mathbb{P}(\langle \text{both drawn balls are blue} \rangle \mid \langle \text{drawn balls have same color} \rangle) \\ &= \mathbb{P}(E \mid C) \\ &= \frac{\mathbb{P}(E \cap C)}{\mathbb{P}(C)} \\ &= \frac{\mathbb{P}(BB)}{\mathbb{P}(\{BB, RR\})} \\ &= \frac{1/5}{1/5 + 1/15} = \frac{3}{4} \end{aligned}$$