

Full Name: _____

Student No: _____

Grade:

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For each answer, **provide a short argument.**

1. (4 points) The function

$$F(x) := \mathbb{P}(X \leq x) = \begin{cases} 0 & \text{if } x < 0, \\ 0.2x^2 & \text{if } 0 \leq x < 1, \\ ax + b & \text{if } 1 \leq x < 2, \\ 1 & \text{if } 2 \leq x. \end{cases}$$

is the cumulative distribution function of a *continuous* random variable X , where a and b are real numbers.

(a) Find the values of a and b , and draw the graph of F .

(b) What is the probability that $0.5 \leq X \leq 2.5$?

(c) Find the probability density function of X .

(d) Find the probability density function of $Y := \sqrt{X}$.

2. (1 point) Let M and N be two independent random variables where $M \sim \text{Bin}(n = 4, p = 0.3)$ and $N \sim \text{Bin}(n = 7, p = 0.3)$. Find the distribution of $X := N + M$.
[Hint: There is an easy solution that does not require any computation.]

3. (1 point) Let U , V and W be independent normal random variables, each with parameters $\mu = 3$ and $\sigma^2 = 4$. What is $\mathbb{P}(U + V \leq W)$? It suffices to write your answer in terms of the cdf $\Phi(z)$ of the standard normal distribution.

4. (1 point) Starting from 2pm when the shop opens, customers arrive at a barber shop according to a Poisson process with a rate of 2 customers per hour. Each customer is served for exactly 30 minutes. If a customer arrives while the shop is empty, they are served immediately. Otherwise, the customer has to wait for their turn. What is the chance that, starting from 2pm, none of the first three customers has to wait before being served?