

Full Name: _____

Student No: _____

Grade:

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For each answer, **provide a short argument.**

1. (5 points) Let X and Y be two random variables with joint pdf

$$f(x, y) := \begin{cases} k & \text{if } x > 0, y > 0 \text{ and } \frac{1}{2}x + \frac{1}{3}y < 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a real number.

- (a) Find the value of k .

- (b) Are X and Y independent?

- (c) What is the probability that $X^2 + Y^2 < 1$?

- (d) Find the expected value of X .

(e) Find the cdf of $Z := \min(X, Y)$.

[Hint: First identify the possible values of Z .]

2. (3 points) Let $U_1, U_2, \dots, U_{1000}$ be independent, identically distributed random variables, each with uniform distribution over the interval $[-\pi, \pi]$. Let

$$S := \sin(U_1) + \sin(U_2) + \dots + \sin(U_{1000}) ,$$
$$M := \frac{1}{1000} \left[\sin(U_1) + \sin(U_2) + \dots + \sin(U_{1000}) \right] .$$

(a) What is the approximate value of M ?

[Hint: Consider $X_i := \sin(U_i)$ for $i = 1, 2, \dots, 1000$.]

(b) Find the (exact) value of $\mathbb{P}(S > 0)$.

(c) Find the approximate value of $\mathbb{P}(S > 5)$.

[Hint: Recall the identity $\sin(\theta)^2 = \frac{1}{2} [1 - \cos(2\theta)]$.]