

Full Name: _____

Student No: _____

Grade:

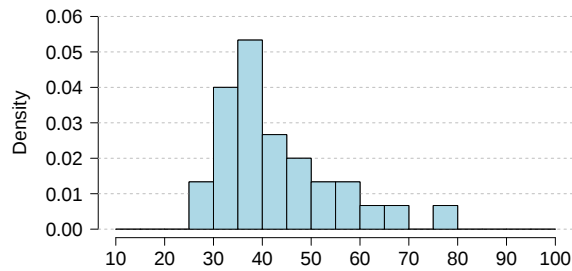
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Read before you start:

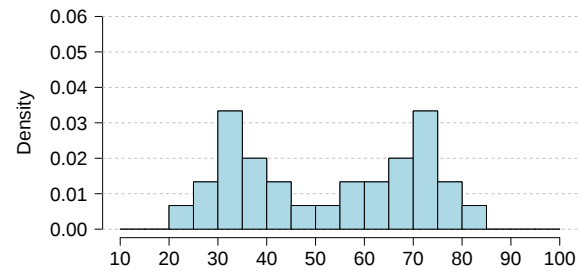
- Please make sure you write your full name and student number.
- The exam consists of 5 questions, each with multiple parts, and a total score of 80 points. All answers require justifications.
- The duration of the exam is 2 hours.

You can use the remainder of this page as scratch paper.

1. (15 points) The following two histograms illustrate the distribution of a numerical variable X within two categories of the same population.



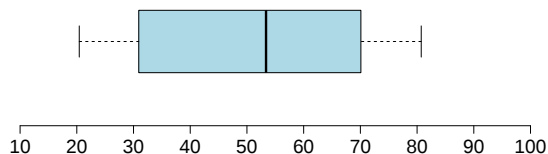
category 1



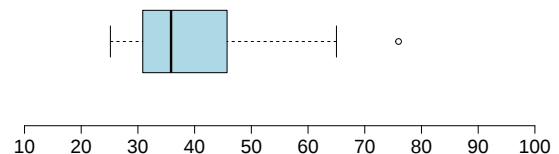
category 2

- (a) Describe the qualitative shapes of the two histograms using terms such as *symmetric*, *right/left-skewed*, *unimodal*, etc.

- (b) The following are the boxplots of the distribution of X over the same two categories. Match the boxplots with the corresponding histograms.



category ?



category ?

- (c) Which of the two categories has a higher proportion of cases with values between 20 and 50?

2. (15 points) We have an ordinary 6-sided die.

(a) What is the probability that in 6 rolls of the die, we get 6 different numbers?

(b) What is the probability that in 6 rolls of the die, we get at least 2 different numbers?

(c) How many times do we have to roll the die so that, with probability at least $\frac{1}{3}$, we encounter at least one 5? (Find the minimum number of times with this property.)

3. (25 points) Nine students are to be assigned to 3 different projects. To this end, the instructor asks each of the students to pick one of the numbers 1, 2, or 3 at random without communicating with the others. The students who pick number i (for $i = 1, 2, 3$) will then be assigned to work together on project number i .
- (a) Choose an appropriate sample space for this random experiment and identify the probability of each possible outcome.
- (b) What is the probability that project number 1 is assigned to at least one student?
- (c) What is the probability that each project is assigned to exactly 3 students?

(d) What is the probability that two friends Jad and Laila end up in the same group?

(e) What is the conditional probability that Jad and Laila end up in the same group if we know that each project is assigned to exactly 3 students?

4. (10 points) We have three boxes each containing 10 balls. Box #1 contains 9 blue balls and 1 red ball, box #2 contains 3 blue balls and 7 red balls, and box #3 contains 4 red balls, 6 yellow balls and no blue balls. We pick a box at random, and draw a ball from it at random.

(a) What is the probability that the drawn ball is blue.

(b) Assuming that the drawn ball is blue, what is the probability that it comes from box #1?

5. (15 points) A jar contains 5 blue balls, 7 red balls, and 8 yellow balls. Jad and Laila play the following game: They draw three balls from the jar at random without replacement. If the drawn balls all have the same colour, Jad wins, and if they all have different colours, Laila wins. If neither of these two possibilities occurs, then the game is a draw (i.e., neither player wins).

(a) Is this game fair? If not, which of the two players has an advantage over the other?

(b) What is the chance that the three drawn balls are blue if we learn that Jad has won?

(c) Suppose the players repeat playing until one of them wins for the first time. What is the chance that the eventual winner is Jad?

You can use this page as extra space for your solutions.