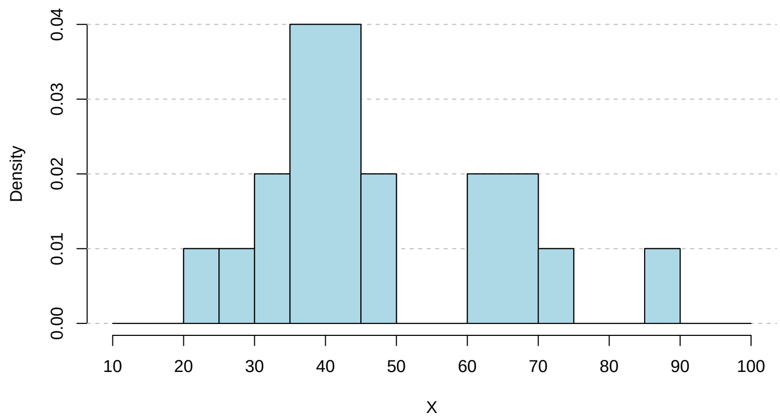
Full Name: ______________________

Student No: ______________________

Grade:

For each answer, **provide a short argument**.

1. (2 points) The following diagram is the histogram of a numerical variable X from a data set.



(a) What percentage of the values are between 60 and 80?

Answer:
25%

Solution: The areas of the rectangles above the interval between 60 and 80 add up to $5/20 = 25\%$ of the total area.

(b) Does 33 lie within the first quartile?

Answer:
Yes!

Solution: The area of the rectangles on the left of 35 is $4/20 = 20\%$ of the total area. Thus, 33 is below the first quartile.

2. (3 points) Two strangers, a carpenter and a plumber, run into each other in the street. Let A denote the event that the two individuals are born on the same day of the year, and let B denote the event that (at least) one of them is born in January. We ignore February 29, and assume that each individual is equally likely to be born on any of the 365 days of the year. January has 31 days.

   (a) What is the probability of A?

   Answer: $\dfrac{1}{365} \approx 0.0027397$

   Solution: Each possible outcome is a pair of days of the year, one as the birthday of the carpenter and one as the birthday of the plumber. The sample space is thus $\Omega := \{1, 2, \dots, 365\}^2$. By assumption, each individual is equally likely to be born on any of the 365 days of the year, hence all the possible outcomes are equally likely.
   In this model,

   $$A = \{(1,1), (2,2), (3,3), \dots, (365,365)\} .$$

   Since all outcomes are equally likely,

   $$\mathbb{P}(A) = \frac{|A|}{|\Omega|} = \frac{365}{365 \times 365} = \frac{1}{365} .$$

   Alternatively, we can intuitively reason as follows: whatever is the birthday of the carpenter, there is a $1/365$ chance that the plumber has the same birthday.

   (b) What is the probability of B?

   Answer: $\frac{21669}{133225} \approx 0.16265$

   Solution: It is easier to consider the probability of the complement. Note

   $$B^{\mathsf{c}} = \{(a,b) \in \Omega : \text{neither } a \text{ nor } b \text{ is in January}\} = \{32, 33, \dots, 365\}^2 .$$

   Therefore,

   $$\mathbb{P}(B) = 1 - \mathbb{P}(B^{\mathsf{c}}) = 1 - \frac{|B^{\mathsf{c}}|}{|\Omega|} = 1 - \frac{(365-31)^2}{365^2} = \frac{21669}{133225} \approx 0.16265$$

   (c) Which is larger: $\mathbb{P}(A \mid B)$ or $\mathbb{P}(B \mid A)$?
   [*Hint:* It is not necessary to compute the two conditional probabilities.]

   Answer: $\mathbb{P}(B \mid A)$

   Solution: Observe that

   $$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} , \qquad \mathbb{P}(B \mid A) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)} .$$

   The numerators are the same. Since $\mathbb{P}(B) > \mathbb{P}(A)$, it follows that $\mathbb{P}(A \mid B) < \mathbb{P}(B \mid A)$.

3. (0 points) On the scale of 0–5, what do you think your grade will be on this quiz?