American University of Beirut

MATH/STAT 233: Probability

2022–2023 Spring

Assignment 1 (due: Monday, February 6)

Suggested (ungraded) warm-up exercises:

- Problems 2.1, 2.3 and 2.7 of Tijms's book. Partial solutions can be found in the back of the book.
- Problems 5, 6, 10, 15 and 19 from Chapter 1 of Chung's book. Partial answers can be found in the back of the book.

Problem 1 (Set difference)**.** Problem 13 from Chapter 1 of Chung's book.

Problem 2 (Inclusion-exclusion)**.**

(a) Problem 20 from Chapter 1 of Chung's book.

(b) Use the first part to give a proof of the combinatorial inclusion-exclusion principle for three sets: If A, B and C are finite sets, then

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \,.$$

Problem 3 (Rock-paper-scissors and the empirical LLN)**.** You are playing rock-paper-scissors with your friend. Each round she wins, you give her a chickpea. Each time you win, she gives you a chickpea. Which of the following statements is true, and which is false? Provide short, informal reasoning or simulation evidence.

(a) After 1000 rounds, you and your friend will have roughly the same number of chickpeas as you used to have.

(b) Within 1000 rounds, the average number of chickpeas each of you wins per round will be roughly the same.

(c) Within 1000 rounds, the proportion of times in which you have more chickpeas than you started with will most likely be around 50%.

You can assume that, at the beginning, each of you has enough chickpeas so that you do not run out of chickpeas.

Problem 4 (Probability models)**.** Problem 2.4 of Tijms's book.

Problem 5 (A coin-flipping game)**.** Two friends, A and B, are playing a game with a coin which has probability $p \in (0, 1)$ of coming up heads in each flip. They repeat flipping the coin until it comes up heads. If the total number of flips is even, A pays \$1 to B; otherwise B pays \$1 to A. (The case $p = {}^1\!/_6$ is a safer variant of the unlimited game of Russian roulette; compare with Example 2.2 in Tijms's book.)

(a) Choose a suitable sample space for this game and assign an appropriate probability to each outcome.

(b) For which value of p is the game fair?

(c) Modify the amount that B pays to A so as to make the game fair in the case $p = {}^1\!/_2$.

Problem 6 (Playful warden)**.** You are a prisoner on an island. Your warden, who is playfully crazy, wants to play the following game with you. There are two boxes and 100 balls, half of which are blue and the remaining half red. You are given the chance to distribute the balls into the two boxes as you wish, as long as (1) no ball is left behind, and (2) each box receives at least one ball. The warden then picks one of the two boxes completely at random, and then picks a ball from that box, again completely at random. If the picked ball is blue, you will be free to go. If not, you will be fed to the sharks. How should you distribute the balls so as to maximize your chance of freedom? Provide an argument showing that your strategy is indeed optimal.