

MATH 300.202
Assignment 10

Due: Wednesday, March 22

Note: *You do not need to hand in your solutions to the starred problems.*

- I. Section 4.1: 4 [Admissible means satisfying conditions of Definition 4.1.1.]
- II. Section 4.2: 14
- III. Section 4.3: 12
- IV. Section 4.4: 5, 19*
[For Prob. 5, use the mathematical formulation of continuous deformation in Definition 4.4.5.]
- V. Section 4.6: 2, 5*, 6, 10, 13
- VI. Section 4.7: 6* [HINT: Use Theorem 4.7.25]
- VII. (a) Let $a_1, a_2, \dots, a_n$ be real numbers with $|a_k| \leq M$ for each k (M is a constant).
Show that if $\left| \frac{a_1 + a_2 + \dots + a_n}{n} \right| = M$, then $a_1 = a_2 = \dots = a_n$.
(b) Show the same thing as in part (a), but this time assume that $a_1, a_2, \dots, a_n$ are complex numbers.
(c) This is the “continuous version” of part (a). Let $f : [a, b] \rightarrow \mathbb{R}$ be a continuous function with $|f(x)| \leq M$ for each $x \in [a, b]$. Show that if $\left| \frac{1}{b-a} \int_a^b f(x) \, dx \right| = M$, then f must be constant.