

MATH 300.202
Assignment 12

NOT to be turned in

- I. Section 5.1: 2(b, d), 7(b, c, e), 14(b), 20, 21
- II. Section 5.3: 2, 4, 5(d), 6, 10
- III. Section 5.5: 2, 9, 12
- IV. Section 5.6: 1(a, d, h), 2, 12, 13, 15
- V. In the class, we showed that

If a power series $\sum_{k=0}^{\infty} a_k z^k$ converges at a point $z_1 \neq 0$, then it also converges at any point in the open disk $\{z : |z| < |z_1|\}$.

Show that the following stronger statement is valid:

If a power series $\sum_{k=0}^{\infty} a_k z^k$ converges at a point $z_1 \neq 0$ and R' is a positive real number strictly smaller than $|z_1|$, then the power series converges *uniformly* on the closed disk $\{z : |z| \leq R'\}$.

[HINT: Follow the proof of the first statement and modify it appropriately.]

[REMARK: Observe that these two statements essentially prove Theorem 5.3.7.]

- VI. [NOTE: We have not covered the prerequisites for this problem but you may still want to try it.]
Let $f_1(z), f_2(z), \dots$ be analytic functions in a disk D and suppose that $f_n(z) \rightarrow f(z)$ *uniformly* in D as $n \rightarrow \infty$. Show that $f'_n(z) \rightarrow f'(z)$ for every z in D .
[HINT: Use Cauchy's formula and Theorem 5.3.8.]

- VI. Section 6.1: 1(a, b, f, h), 2, 3(a, c, e), 4
- VII. Section 6.3: 2, 3, 4, 8, 13