MATH 300.202

Assignment 3

Due: Wednesday, January 25

I. Section 1.6: 17, 18, 20

II. Section 2.1: 4, 6, 10, 12

III. Section 2.2: 7(c, e, f), 12, 14

IV. Find each of the following limits or specify if the limit does not exist.

(a) $\lim_{z\to \mathrm{i}} z^3 + 3\mathrm{i}z + 1 - 2\mathrm{i}$

(b) $\lim_{z\to -\mathrm{i}} \dfrac{z^4 - 1}{z + \mathrm{i}}$

(c) $\lim_{z\to 1+\mathrm{i}} \dfrac{z^2 + z - 1 - 3\mathrm{i}}{z^2 - 2z + 2}$

(d) $\lim_{z\to 0} \dfrac{z^2}{z\overline{z}}$

V. Determine *where* the following functions are continuous.

(a) $f(z) \coloneqq \operatorname{Arg}(z)$

(b) $f(z) \coloneqq z + \overline{z}$

(c) $f(z) \coloneqq \frac{1}{|z|-1}$

(d) $f(z) \coloneqq \begin{cases} \frac{z^3+1}{z+1} & \text{if } z \neq 1, \\ 1 & \text{otherwise.} \end{cases}$

VI. Show that the *geometric inversion* mapping $f(z) \coloneqq 1/\overline{z}$ (a.k.a., reflection in the unit circle) maps every line not passing through the origin onto a circle passing through the origin excluding the origin itself. (For simplicity, you may assume that the line does not intersect the unit circle.) [HINT: Consider the images of two points z_0 and z on the line ℓ, chosen such that the vector z_0 is perpendicular to the line ℓ and z is distinct from z. Look for similar triangles and determine the locus of $f(z)$ as z varies over ℓ.]

Last update: January 18, 2017