MATH 300.202

# Assignment 5

Due: Wednesday, February 8

I. Section 2.5: 2, 4, 6, 8, 12, 18

II. For each of the following cases, use a computer to plot level curves of $u$ and $v$ on the same plane (see examples on the course website) and visually verify that they intersect each other at right angles. Explain your observations.

(a) $u(x, y) \coloneqq x^2 - y^2$ and $v(x, y) \coloneqq 2xy$.

(b) $u(x, y) \coloneqq \mathrm{e}^x \cos y$ and $v(x, y) \coloneqq \mathrm{e}^x \sin y$.

(c) $u(x, y) \coloneqq \frac{x}{x^2+y^2}$ and $v(x, y) \coloneqq \frac{-y}{x^2+y^2}$.

[You do not need to hand in this exercise.]

III. Show that the function $p(x, y) \coloneqq \ln |z|$ is harmonic in the domain $\mathbb{C} \setminus \{0\}$.

IV. Show that $q(x, y) \coloneqq \operatorname{Arg}(z)$ is a harmonic conjugate of $p(x, y)$ from the previous problem, defined in the domain $\mathbb{C} \setminus \{x + 0\mathrm{i} : x \leq 0\}$ (i.e., the entire plane excluding the ~~non-negative~~ non-positive real axis). Moreover, show that $p(x, y)$ does not have a harmonic conjugate defined in the domain $\mathbb{C} \setminus \{0\}$.

Last update: February 5, 2017