

Full Name: \_\_\_\_\_

Student No: \_\_\_\_\_

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| Grade: |
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1. (4 points) Determine whether each of the following statements is TRUE or FALSE.  
[No explanation is needed.]

(a) The complex exponential function maps vertical lines to rays starting at (but not containing) the origin, and it maps the horizontal lines to circles centered at the origin.

Answer:

(b) The function  $f(x + iy) := y + ix$  is nowhere differentiable.

Answer:

(c) If  $f(z)$  is analytic with  $f'(z) = 0$  in an open set  $A$  (not necessarily connected), then  $f$  is necessarily constant in  $A$ .

Answer:

(d) The function  $u(x, y) := e^{-y} \sin x$  is harmonic in the entire plane.

Answer:

2. (4 points)

(a) Find all the points  $z = x + iy$  at which the function

$$f(z) := x^2y - y + x + i(xy^2 + y + 1)$$

is differentiable.

(b) Identify where  $f$  is analytic.

3. (2 points) Show that if  $f(z)$  is analytic in the entire plane and its image is contained in the hyperbola  $\{(x, y) : xy = 1\}$ , then  $f$  has to be constant.