

Full Name: _____

Student No: _____

Grade:

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1. (4 points) Determine whether each of the following statements is TRUE or FALSE.
[No explanation is needed.]

(a) $\text{Log}(z_1 z_2) = \text{Log } z_1 + \text{Log } z_2$ for every $z_1, z_2 \in \mathbb{C} \setminus \{0\}$.

Answer:

(b) $e^{\text{Log } z} = z$ for every $z \in \mathbb{C} \setminus \{0\}$, and $\text{Log } e^z = z$ for every $z \in \mathbb{C}$.

Answer:

(c) The coefficient of $1/z$ in the partial fraction decomposition of $R(z) := \frac{z-1}{z^2(z+1)}$ is 2.

Answer:

(d) If $z \neq 0$ and $\alpha \in \mathbb{R}$, then all the values of z^α are on a circle centered at the origin, and all the values of $z^{i\alpha}$ are on a ray emitting from the origin.

Answer:

2. (2 points) Argue that $u(x, y) := (\ln |z|)^2 - (\operatorname{Arg} z)^2$ (where $z = x + iy$) is harmonic in the domain $D^* := \mathbb{C} \setminus \{x + iy : x \leq 0 \text{ and } y = 0\}$.

[HINT: Recall that the real and imaginary parts of an analytic function are harmonic.]

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3. (4 points) Determine the domain of analyticity for $f(z) := \text{Log}(e^{iz} + 1)$. Sketch the set of points at which f is not analytic and compute $f'(0)$.