Full Name: ____________________

Student No: ____________________

Grade:

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1. (4 points) Determine whether each of the following statements is TRUE or FALSE. [No explanation is needed.]

   (a) $\displaystyle\oint_{|z|=1} \overline{z}\,\mathrm{d}z = \oint_{|z|=1} \frac{1}{z}\,\mathrm{d}z.$

   Answer:

   (b) If $f(z)$ is analytic at every point in the closed disk $\overline{D} := \{z : |z| \leq 1\}$ and $|f(z)|$ takes a maximum value somewhere in $\overline{D}$, then f is necessarily constant in $\overline{D}$.

   Answer:

   (c) If $f(z)$ has an anti-derivative in a domain D, then $f(z)$ is analytic in D.

   Answer:

   (d) $\displaystyle\int_\gamma \frac{z^{16} - 2016}{z^{2017}}\,\mathrm{d}z = 0$ for every closed contour γ that does not pass through the origin.

   Answer:

2. (4 points) Find the value of the following integrals.

(a) $\displaystyle\int \frac{\mathrm{Re}(z)}{z}\,\mathrm{d}z$ from $-3\mathrm{i}$ to $3\mathrm{i}$ along the circle $|z| = 3$ traversed counter-clockwise.

(b) $\displaystyle\oint_{|z|=5} \frac{2\cos z}{z^2(2z-\pi)}\,\mathrm{d}z$

3. (2 points) Let $f(z)$ be an entire function and suppose that $|f(z)| \leq 1000\,|e^z|$ whenever $|z| \geq 10$. Show that $f(z) = c\,e^z$ for some constant $c \in \mathbb{C}$. [HINT: consider $f(z)/e^z$.]