Full Name: _____

Student No: _____

Grade:

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1. (4 points) Determine whether each of the following statements is TRUE or FALSE. [No explanation is needed.]

   (a) If the series $\sum_{k=0}^{\infty} a_k z^k$ converges at $z = 2\mathrm{i}$, then its radius of convergence is at least 2.

   Answer:

   (b) The Taylor series of e^z about $z = 0$ converges uniformly over the unit disk.

   Answer:

   (c) The function $\operatorname{Log} z$ has a Laurent expansion about $z = 0$ that is convergent in the punctured unit disk $0 < |z| < 1$.

   Answer:

   (d) The function $\dfrac{1 - \cos z}{z^3}$ has a pole of order 3 at $z = 0$.

   Answer:

2. (3 points) Find a Laurent expansion of $g(z) \coloneqq \dfrac{2(z+2)}{z^2-1}$ about $z=-2$ that is convergent at $z=0$.

3. (3 points) Show that the series $\sum_{k=0}^{\infty} \frac{z^k}{1+z^{2k}}$ converges if and only if z is *not* on the unit circle.